Mechanistic Analysis and Design of Pavements

- Introduction
- Stresses in Multi-Layer Systems
  - Stresses in Flexible Pavements
  - Stresses in Rigid Pavements
- Characterization of Geomaterials
  - Constitutive Behavior
  - Stress Path Testing
- Characterization of Asphaltic Materials
  - Viscoelastic Behavior
  - Binder and Mix Characterization
- Traffic
  - ESAL Concept
  - Axle Load Spectra
- Design Methods
  - AASHTO Method
  - Asphalt Institute (AI)
  - TxDOT Method
- Distresses in Pavements
  - New Mechanistic Empirical Design Guide

Software Tools:
- WinJULEA
- KENLAYER
- AASHTO T-307
- NCHRP 1-28A
- Superpave Binder Tests
- AASHTO Mix Tests
- Flexible Pavement Design
- Rigid Pavement Design
- FPS21 Software
- MEPDG Software
Factors Influencing the Resilient Response of Granular Materials

<table>
<thead>
<tr>
<th>Factor</th>
<th>Stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Broader Grading</td>
<td>minor ↑</td>
</tr>
<tr>
<td>Fines</td>
<td>↓</td>
</tr>
<tr>
<td>Larger Size</td>
<td>↑</td>
</tr>
<tr>
<td>More Rough/Angular</td>
<td>↑</td>
</tr>
<tr>
<td>Density</td>
<td>↑</td>
</tr>
<tr>
<td>Moisture</td>
<td>major ↓</td>
</tr>
</tbody>
</table>
Constitutive Model for Geomaterials

- **Stress Sensitivity**
  - *Hardening Behavior*
    - Densification under load
  - *Softening Behavior*
    - Loss of strength with increasing either load magnitude or number of load repetition

- **Nonlinearity**
  - Nonlinear response in stress strain curve particularly near plastic yield.

- **Anisotropy**
  - Directional dependency of material properties.
Movement of the Column of Confinement in the UAB

\[ E = k_1 \theta^{k_2} \]

\[ E = Pa k_1 \left( \frac{\theta}{Pa} \right)^{k_2} \left( \frac{\tau_{oct}}{Pa} + 1 \right)^{k_3} \]
Distribution of Vertical Stresses Induced by the Wheel Load in the Base Layer
Distribution of Horizontal Stresses Induced by the Wheel Load in the Base Layer
Nonlinear Distribution of Vertical Modulus ($E_y$) in the Base Layer
Nonlinear Distribution of Horizontal Modulus ($E_x$) in the Base Layer

<table>
<thead>
<tr>
<th>E HORZ</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 264.17</td>
<td>264.17</td>
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<tr>
<td>264.17 - 512.22</td>
<td>512.22</td>
</tr>
<tr>
<td>512.22 - 760.26</td>
<td>760.26</td>
</tr>
<tr>
<td>760.26 - 1008.31</td>
<td>1008.31</td>
</tr>
<tr>
<td>1008.31 - 1256.36</td>
<td>1256.36</td>
</tr>
<tr>
<td>1256.36 - 1504.41</td>
<td>1504.41</td>
</tr>
<tr>
<td>1504.41 - 1752.45</td>
<td>1752.45</td>
</tr>
<tr>
<td>1752.45 - 2000.50</td>
<td>2000.50</td>
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<tr>
<td>2000.50 - 2248.55</td>
<td>2248.55</td>
</tr>
<tr>
<td>2248.55 - 2496.59</td>
<td>2496.59</td>
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<tr>
<td>2496.59 - 2744.64</td>
<td>2744.64</td>
</tr>
<tr>
<td>&gt; 2744.64</td>
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</table>
Characterization of Unbound Aggregate Systems

- **Constitutive Model**
  
  - The proposed model should be able to capture the following characteristics of particulate materials:
    - Stress sensitivity
    - Nonlinearity
    - Anisotropy

- **Stress Path Testing (Loading Protocol in the Laboratory)**
  
  - Simulation of moving wheel load in the lab
  - Sequence of stress regimens or stress history
    - (Extension-Compression-Extension)
  - Stress Ratios (slope of stress path)
  - Stress Magnitude (stress path length)
Measures of Nonlinearity

Method One:

\[ M_n = \frac{\varepsilon_f}{\varepsilon_r} \]

\( \varepsilon_r \): Resilient strain

(maximum axial strain of the linear part of stress-strain curve)

\( \varepsilon_f \): Strain at failure

Method Two:

\[ M_n = \frac{q_f}{q_r} \]

\( q_r \): Deviatoric stress

(maximum deviatoric stress of the linear part of stress-strain curve)

\( q_f \): Deviatoric stress at failure
Sources of Anisotropy

Inherent Anisotropy

✓ Particle arrangement and optimum packing is a function of aggregate shape properties.

✓ Maximum dimension tend to align in the horizontal plane.
Characterization of the Particle Orientation

Direction of the Longest Axis

Normal to the Interface

Branch Vector

Normal to the Polygon Representing Air Void Trapped between Particles
Orientation distribution of equi-dimensional single size particles (no inherent anisotropy)

Orientation distribution of mixed size particles

Orientation distribution of single size particulate system with flat and elongated particles (significant inherent anisotropy)
Microstructure Distribution of Particle Contacts

- Microstructure Tensor:

\[
M_{ij} = \int_{\Omega} E(l) l_i l_j d\Omega
\]

- Probability density function of microstructure distribution:

\[
E(l) = \frac{15}{8\pi} \left( M_{ij} - \frac{1}{5} \delta_{ij} \right) l_i l_j
\]

\[
E(l) = \frac{1}{4\pi} \left( 1 + M_{ij} l_i l_j \right)
\]
Non-homogenous distribution of particle orientations with preferred orientation in the direction of deposition/compaction.

Upon compaction, aggregate particles subjected to compaction rollers tend to rearrange themselves in a way to increase particle contacts and reduce air voids to achieve maximum density.
Impact of Aggregate Angularity and Texture on the Level of Anisotropy (Ashtiani, 2008)
Impact of Aggregate Form and Texture on The Level of Anisotropy (Ashtiani, 2008)

Measure of Aggregate Texture (Ta)

More Spherical

Rough Texture

Gxx/Gxy

< 0.44

0.44 - 0.48

0.48 - 0.52

0.52 - 0.56

0.56 - 0.60

0.60 - 0.64

0.64 - 0.68

0.68 - 0.72

0.72 - 0.76

0.76 - 0.80

> 0.80
Sources of Anisotropy (Cont.)

Rotation of Principal Stresses

- As a wheel load passes over a point in the UAB the stress ratio (major to minor principal stress ratio) and orientation of principal stresses changes.
- At some radial distance away from the wheel load, the magnitude of horizontal stress becomes higher than the vertical stress.
The orientation of the principal plane changes as the wheel load passes over the pavement.

At some radial distance away from the wheel load the significance of load induced anisotropy dramatically diminishes.

\[ \theta_{\sigma_1, \sigma_1} = \frac{1}{2} \tan^{-1} \left( \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \right) \]
Understanding the Mechanical Responses of Inverted Pavements (Ashtiani, TRB 2016)

<table>
<thead>
<tr>
<th>Layer</th>
<th>Type</th>
<th>Modulus (psi)</th>
<th>Poisson's Ratio</th>
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</thead>
<tbody>
<tr>
<td>Asphalt (AC)</td>
<td>Linear Isotropic</td>
<td>$E_{AC}=500,000$</td>
<td>$\nu = 0.35$</td>
</tr>
<tr>
<td>UAB</td>
<td>Nonlinear Anisotropic</td>
<td></td>
<td>Modular Ratio $= 0.3$</td>
</tr>
<tr>
<td>Cement Treated Base</td>
<td>Linear Isotropic</td>
<td></td>
<td>$E_{CTB}=200,000$</td>
</tr>
<tr>
<td>Subgrade</td>
<td>Nonlinear Isotropic</td>
<td></td>
<td>$E_{SG}=6,000$</td>
</tr>
</tbody>
</table>

Diagram:
- Inverted Design: 4" AC, 12" UAB, 8" CTB
- Traditional Design: 4" AC, 8" CTB, 12" UAB

Legend:
- 100 psi pressure applied.
Pavement Responses in Traditional vs. Inverted Sections (Ashtiani, TRB 2016)

Vertical stresses at the top of the UAB

Shear stresses at the top of the UAB
Orientation of the Principal Plane
Inverted vs. Traditional Design (Ashtiani, TRB 2016)
Moving Wheel Load Stresses
Field Measurements

Station (X), ft

Compression
Vertical
Horizontal

Extension

LFS Section
10 in. Asphalt
29.6 in. P209

DUAL GEAR
TRANSVERSE OFFSET #2
TRAFFIC PATH SOUTH
Stress History

The Testing protocol should be able to simulate the actual Extension - Compression - Extension nature of stress sequence observed in the approaching and departing wheel load.

![Diagram](image-url)

- **Extension** - $s_x > s_y$
- **Compression** - $s_x < s_y$
- **Extension** - $s_x > s_y$

**Single Stress Path Test**

**Multiple Stress Path Test**

- Loading: $s_1 < s_3$
- Unloading: $s_1 > s_3$

Radial Distance $R$
Actual Field Measurement of Moving Wheel Stress Path

Compression

Extension

$p^* = (2\sigma_h + \sigma_v)/3$, psi
Stress induced anisotropy

- Toyoura sand
- SLB sand
- Ticino sand
- Hime gravel
- Chiba gravel
- Nerima gravel

Inherent anisotropy

Perfectly isotropic material
Cross Anisotropic Constitutive Equation

**Generalized Hook’s Law**

\[ \varepsilon_{ij} = C_{ijkl} \sigma_{kl} \]

- \( E_y \) = Vertical elastic modulus
- \( E_x \) = Horizontal elastic modulus
- \( G_{xy} \) = Shear modulus
- \( \nu_{xy} \) = Vertical Poisson ratio
- \( \nu_{xy} \) = Horizontal Poisson ratio
- \( \sigma_x \) = Radial stress
- \( \sigma_y \) = Vertical stress
- \( \varepsilon_x \) = Radial strain
- \( \varepsilon_y \) = Vertical strain
Independent Elastic Constants Required to Characterize Various Types of Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>No. of Independent Elastic Constants</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Anisotropic Material</td>
<td>81</td>
</tr>
<tr>
<td>Anisotropic material considering symmetry of stress strain tensor ($\sigma_{ij} = \sigma_{ji}$, $\varepsilon_{ij} = \varepsilon_{ji}$)</td>
<td>36</td>
</tr>
<tr>
<td>Anisotropic material considering elastic energy considerations</td>
<td>21</td>
</tr>
<tr>
<td>General Orthotropic Material</td>
<td>9</td>
</tr>
<tr>
<td>Orthotropic Material with transverse isotropy (cross-anisotropic material)</td>
<td>5</td>
</tr>
<tr>
<td>Isotropic material</td>
<td>2</td>
</tr>
</tbody>
</table>
Cross Anisotropic Elastic Properties

Five Elastic Properties needed to characterize the material

Plane of Isotropy
Choice of Material Model (ICAR Protocol)

✓ Resilient Modulus in Vertical Direction

\[
E_y = K_1 \left( \frac{\theta}{P_a} \right)^{K_2} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{K_3}
\]

✓ Resilient Modulus in Horizontal Direction

\[
E_x = K_4 \left( \frac{\theta}{P_a} \right)^{K_5} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{K_6}
\]

✓ Shear Modulus in xy plane

\[
G_{xy} = K_7 \left( \frac{\theta}{P_a} \right)^{K_8} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{K_9}
\]
Case Study
Comparison between Material Models
Critical Pavement Responses
4 Inch Asphalt Pavement

4” AC
12” UAB
Soft Subgrade

<table>
<thead>
<tr>
<th>Asphalt</th>
<th>Nonlinear Isotropic</th>
<th>$K_1=28000$, $k_2=0.1$ and $k_3=0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>UAB</td>
<td>Nonlinear Anisotropic</td>
<td>$K_1=3500$, $k_2=0.5$, $k_3=-0.1$ &amp; AR=0.2</td>
</tr>
<tr>
<td>Natural Subgrade</td>
<td>Nonlinear Anisotropic</td>
<td>$K_1=206$, $k_2=0.0$, $k_3=-0.3$ &amp; AR=0.5</td>
</tr>
</tbody>
</table>

Strain
- AC Tensile Strain
- SG Compressive Strain

<table>
<thead>
<tr>
<th>Nonlinear Anisotropic</th>
<th>Linear Anisotropic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonlinear Isotropic</td>
<td>Linear Isotropic</td>
</tr>
</tbody>
</table>

100 psi
Critical Pavement Responses
2 Inch Asphalt Pavement

<table>
<thead>
<tr>
<th>Asphalt</th>
<th>Nonlinear Anisotropic</th>
<th>$K_1$</th>
<th>$k_2$</th>
<th>$k_3$</th>
<th>AR</th>
</tr>
</thead>
<tbody>
<tr>
<td>UAB</td>
<td>Nonlinear Anisotropic</td>
<td>28000</td>
<td>0.1</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>Natural Subgrade</td>
<td>Nonlinear Anisotropic</td>
<td>3500</td>
<td>0.5</td>
<td>-0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>Asphalt</td>
<td>Nonlinear Isotropic</td>
<td>2070</td>
<td>0.0</td>
<td>-0.3</td>
<td>0.5</td>
</tr>
</tbody>
</table>

AC Tensile Strain

SG Compressive Strain
Pavement Response Analysis
Vertical Stress in Base Layer

**Linear Isotropic**

\[ E_x = E_y \]

**Nonlinear Anisotropic**

\[ E_x = 0.3 \, E_y \]
Pavement Response Analysis
Horizontal Stress in Base Layer

Linear Isotropic
\[ E_x = E_y \]

Nonlinear Anisotropic
\[ E_x = 0.3 E_y \]
Characterization of Geomaterials in the Laboratory
Monotonic/Cyclic Axial Load (haversine load shape)

Constant/Variable Cell Pressure (air or liquid)

Top Platen
Pressure Chamber
Membrane
Embedded Stud with Screw Fixing
Base
Sealing Ring

Axial Strain Measurement
Radial Strain Measurement

Cylindrical Specimen

Axial Load Sensor
Bushing
Typical Response of Aggregate Layers

Deviatoric Stress (kPa) vs. Axial Strain ($\mu e$)

Confining Stress = 100 kPa

Unbound Slate Waste

- $N=1-3$
- 100
- 1000
- 20000
- 80000
Resilient vs. Elastic Modulus

AASHTO’s Definition:
“A measure of the elastic property of soil recognizing certain nonlinear characteristics.”
stress state:1
stress state:1
sweep number:1
sweep number:1
filename:6
filename:6
test temperature (°C): 0.0
strain reference: current
test temperature (°C): 0.0
strain reference: current
test frequency (Hz): 0.67
sweep cycles: 25
pulse width (msec): 1500
cycle count: 12
pulse width (msec): 1500
cycle count: 12

Average Axial Strain
1.147 (με) per div

Time
298.8 (msec) per div

21000 23998 26976 29964 32952 35940
stress state: 2
test temperature (°C): 0.0
strain reference: current
sweep number: 1
test frequency (Hz): 0.67
sweep cycles: 25
filename: 2
pulse width (msec): 1500
cycle count: 15

Average Axial Strain
2.861 (με) per div

Time
298.8 (msec) per div

Graphical display options:
- Est
- Plots
- Scales
- Reference
- Next
- State
- Filter
- Zoom
- Analyse
Resilient Response in Unbound Granular Materials

- In most of the design methods, it is assumed that the permanent strain for a well designed granular material decreases to a very small rate, and it is sufficient to describe the unbound layer behavior using elastic properties.

- Some of the design methods assume the resilient modulus to be constant (linear stress-strain behavior), while others express it as a function of stress level.

- Resilient or elastic properties of unbound layers are determined using a repeated load triaxial test results.

\[
M_r = \frac{\Delta(\sigma_1 - \sigma_3)}{\varepsilon_{1,r}}
\]

- \(\sigma_1\) and \(\sigma_3\) = Major and minor principal stresses
- \(\varepsilon_{1,r}\) = Major principal resilient strain
- \(\mu\) = Poisson’s ratio
- \(\varepsilon_{3,r}\) = Minor principal resilient strain
Stress Sensitivity of the Resilient Modulus Material Models - Choice of Material Model

$M_r$ is a function of the state of stresses:

\[
M_r = k_1 P_a \left( \frac{\sigma_3}{P_a} \right)^{k_2}
\]

\[
M_r = k_1 P_a \left( \frac{\theta}{P_a} \right)^{k_2}
\]

\[
M_r = k_1 P_a \left( \frac{\sigma_d}{P_a} \right)^{k_2}
\]

\[
M_r = k_1 P_a \left( \frac{\tau_{oct}}{P_a} \right)^{k_2}
\]

\[
M_r = k_1 P_a \left( \frac{\theta}{P_a} \right)^{k_2} \left( \frac{\sigma_d}{P_a} \right)^{k_3}
\]

- $\sigma_3$: Confining stress,
- $P_a$: Atmospheric pressure, constant
- $\theta$: Sum of principal stresses
- $\sigma_d$: Deviatoric stress ($\sigma_1 - \sigma_3$),
- K-values: Material constants, $k_2$ positive, $k_3$ negative
Octahedral stresses are the normal and shear stresses that are acting on some specific planes inside the stressed body, the octahedral planes.

If we consider the principal directions as the coordinate axes, then the plane on which the normal vector forms equal angles with the coordinate system is called octahedral plane. There are eight such planes forming an octahedron as shown in this slide.
Typical Results of Resilient Modulus Test
Comparisons of Crushed Stone and Sand Mixes

![Graph showing comparisons of resilient moduli for crushed stone and sand mixes.](image-url)
Effect of Degree of Saturation on Resilient Modulus ($M_r$)
Choice of Material Model

NCHRP 1-28 A study made some recommendations to improve $M_r$ testing:

- Sample size dependent on maximum aggregate size;
- Modified loading times;
- New model for $M_r$:

$$M_r = k_1 P_a \left( \frac{\theta - 3k_6}{P_a} \right)^{k_2} \left( \frac{\tau_{oct}}{P_a} - k_7 \right)^{k_3}$$

where $k_1, k_2 \geq 0, k_3, k_6 \leq 0, k_7 \geq 1$
Choice of Material Model
New MEPDG Model

\[ E_y = Pa \left( \frac{I_1}{Pa} \right)^{k_2} \left( \frac{\tau_{oct}}{Pa} + 1 \right)^{k_3} \]

- **Hardening Term**
- **Softening Term**

\[ k_1 = 2500 \]

\[ k_2 = 0.5 \]

\[ k_3 = -0.3 \]
Softening Term = \left(\frac{\tau_{oct}}{p_a}\right)^{k_3}

![Graph showing the softening term as a function of octahedral shear stress with various values of k3.]
Softening Term = \left( \frac{\tau_{oct}}{p_a} + 1 \right)^{k_3}

- \( k_3 = 1 \)
- \( k_3 = 0.7 \)
- \( k_3 = 0.5 \)
- \( k_3 = 0.2 \)
- \( k_3 = 0 \)
- \( k_3 = -0.2 \)
- \( k_3 = -0.5 \)
- \( k_3 = -0.7 \)
- \( k_3 = -1 \)

Octahedral Shear Stress (psi)
Characterization of Geomaterials in Pavement Design
Typical Characterization of UAB in Pavement Analysis-Design

- Stress-sensitive (nonlinear) elastic - pavement structure

\[ E_{UAB} = K_1 \theta^k \]

\[ \theta = \sigma_x + \sigma_y + \sigma_z + \gamma z (1 + 2k_0) \]

\[ E_{sub} = K_1 + K_3 (K_2 - \sigma_d) \]

\[ \sigma_d = \sigma_1 - 0.5(\sigma_2 + \sigma_3) + \gamma z (1 - k_o) \]

- Limitations
  - Horizontal stiffness variation not considered.
  - Effect of tensile radial stresses introduces a negative \( \theta \) which must be accounted for - not realistic.
  - Correction attempts.
Sensitivity Analysis of K-θ Model

\[ M_R = k_1 (\theta)^{k_2} \]

- Sensitivity of bulk stress model to \( k_1 \) parameter (\( k_2 = 1 \))
- Sensitivity of bulk stress model to \( k_2 \) parameter (\( k_1 = 4000 \))
Softening Behavior in Fine Grained Soils

Modeling the Behavior of Fine Grained Soils:

\[ M_R = K_1 + K_3(K_2 - \sigma_d) \]  \text{ When } \sigma_d < K_2

\[ M_R = K_1 - K_4(\sigma_d - K_2) \]  \text{ When } \sigma_d > K_2

- \( M_R \): Resilient modulus
- \( \sigma_d \): Deviator stress (\( \sigma_1 - \sigma_3 \))
- \( K_1, K_2, K_3, K_4 \): Experimentally derived material constants
Typical resilient modulus-deviator stress relationship for fine grained materials by Thompson and Elliot (1985).

The value of the breakpoint in the bilinear curve ($K_1$) is a good indicator of resilient behavior.

$K_2$, $K_3$ and $K_4$ display a smaller influence on the resilient Modulus.
Example

Repeated triaxial tests on a granular material intended to be used as a base layer in an asphalt pavement:

<table>
<thead>
<tr>
<th>Sequence No.</th>
<th>Confining $\sigma_3$ (kPa)</th>
<th>Max Axial $\sigma_{max}$ (kPa)</th>
<th>Cyclic (kPa)</th>
<th>Constant 0.1$\sigma_{max}$ (kPa)</th>
<th>$M_r$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.7</td>
<td>20.7</td>
<td>18.6</td>
<td>2.1</td>
<td>69,527.30</td>
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<td>37.3</td>
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<td>20.7</td>
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<td>68.9</td>
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<td>68.9</td>
<td>137.9</td>
<td>124.1</td>
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<td>215,703.02</td>
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<tr>
<td>9</td>
<td>68.9</td>
<td>206.8</td>
<td>186.1</td>
<td>20.7</td>
<td>231,037.44</td>
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<tr>
<td>10</td>
<td>103.4</td>
<td>68.9</td>
<td>62.0</td>
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<td>341,371.40</td>
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<tr>
<td>11</td>
<td>103.4</td>
<td>103.4</td>
<td>93.1</td>
<td>10.3</td>
<td>319,456.68</td>
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<tr>
<td>12</td>
<td>103.4</td>
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<td>186.1</td>
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</tbody>
</table>

- Plot $M_r$ versus bulk stress ($\theta$) and octahedral stress ($\tau_{oct}$). Discuss the results.
- Determine appropriate values for the constants $k_1$, $k_2$ and $k_3$ by fitting previous equations to the plots.
Solution

\[
M_r = k_1 P_a \left( \frac{\theta}{P_a} \right)^{k_2} \left( \frac{\tau_{oct}}{P_a} \right)^{k_3}
\]

\[
\tau_{oct} = \frac{1}{3} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}
\]

\[
\theta = \sigma_1 + 2\sigma_3
\]

Since \(\sigma_2 = \sigma_3\), then:

\[
\tau_{oct} = \frac{\sqrt{2}}{3} \sigma_d
\]
Solution- cont.

Solution: relationship between $M_r$ and $\theta$.

$M_r$ increases as a function of $\theta$
Solution-Cont.

Relationship between $M_r$ and $\tau_{\text{oct}}$.

Larger $\tau_{\text{oct}}$, smaller $M_r$
Solution- Cont.

Regression on the given data produces the following coefficients:

- From: \( M_r = k_1 P_a \left( \frac{\Theta}{P_a} \right)^{k_2} \)
  \( k_1 = 709.8 \)
  \( k_2 = 0.985 \)

- From: \( M_r = k_1 P_a \left( \frac{\Theta}{P_a} \right)^{k_2} \left( \frac{\tau_{oct}}{P_a} \right)^{k_3} \)
  \( k_1 = 306.06 \)
  \( k_2 = 1.415 \)
  \( k_3 = -0.467 \)
Choice of Material Model (ICAR Protocol)

✓ Resilient Modulus in Vertical Direction

\[ E_y = K_1 P_a \left( \frac{\theta}{P_a} \right)^{K_2} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{K_3} \]

✓ Resilient Modulus in Horizontal Direction

\[ E_x = K_4 P_a \left( \frac{\theta}{P_a} \right)^{K_4} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{K_6} \]

✓ Shear Modulus in xy plane

\[ G_{xy} = K_7 P_a \left( \frac{\theta}{P_a} \right)^{K_8} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{K_9} \]
Anisotropy of Aggregate Systems

\[ E_y = K_1 P_a \left( \frac{\theta}{P_a} \right)^{K_2} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{K_3} \]

\[ E_x = K_4 P_a \left( \frac{\theta}{P_a} \right)^{K_5} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{K_6} \]

![Graph showing vertical and horizontal moduli vs. deviator stress](image-url)
<table>
<thead>
<tr>
<th>General Soil Type</th>
<th>USC Soil Type</th>
<th>CBR Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse-grained soils</td>
<td>GW</td>
<td>40 - 80</td>
</tr>
<tr>
<td></td>
<td>GP</td>
<td>30 - 60</td>
</tr>
<tr>
<td></td>
<td>GM</td>
<td>20 - 60</td>
</tr>
<tr>
<td></td>
<td>GC</td>
<td>20 - 40</td>
</tr>
<tr>
<td></td>
<td>SW</td>
<td>20 - 40</td>
</tr>
<tr>
<td></td>
<td>SP</td>
<td>10 - 40</td>
</tr>
<tr>
<td></td>
<td>SM</td>
<td>10 - 40</td>
</tr>
<tr>
<td></td>
<td>SC</td>
<td>5 - 20</td>
</tr>
<tr>
<td>Fine-grained soils</td>
<td>ML</td>
<td>15 or less</td>
</tr>
<tr>
<td></td>
<td>CL LL &lt; 50%</td>
<td>15 or less</td>
</tr>
<tr>
<td></td>
<td>OL</td>
<td>5 or less</td>
</tr>
<tr>
<td></td>
<td>MH</td>
<td>10 or less</td>
</tr>
<tr>
<td></td>
<td>CH LL &gt; 50%</td>
<td>15 or less</td>
</tr>
<tr>
<td></td>
<td>OH</td>
<td>5 or less</td>
</tr>
</tbody>
</table>
Plastic Deformation in Geomaterials
Plastic Response

- Estimate the plastic component of the strain in granular materials: important for quantifying permanent deformation.
- There are two approaches to model the plastic response of unbound aggregate bases/sub-bases and subgrades:
  - Use a 3-D strain-behavior model of the aggregates based on plasticity theory;
  - Use laboratory experiments to obtain a one-dimensional relationship between stress level, number of loading cycles and cumulative permanent strain.
Shakedown Theory

- Post-compaction
- Plastic shakedown limit
- Plastic creep limit
- Stiffening response
- Incremental collapse
## Empirical Relationships for Plastic Deformations

<table>
<thead>
<tr>
<th>Model</th>
<th>Expression</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Khedr (1985)</td>
<td>( \frac{\varepsilon_{1,p}}{N} = A_1 N^{-b} )</td>
<td>( \varepsilon_{1,p} ): 1-D permanent strain</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( N ): number of cycles</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( A_1, b ): material constants</td>
</tr>
<tr>
<td>VESYS (1977)</td>
<td>( \varepsilon_{1,p}(N) = \varepsilon_{r,200} N^{-\alpha} )</td>
<td>( \varepsilon_{1,p}(N) ): Plastic strain due to a single application at the ( N^{th} ) cycle</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \varepsilon_{r,200} ): Resilient strain at 200 cycles</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \nu ): Proportionality constant</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \alpha ): Material constant</td>
</tr>
</tbody>
</table>
### Empirical Relationships of Plastic Deformations, Cont.

<table>
<thead>
<tr>
<th>Model</th>
<th>Expression</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tseng and Lytton (1989)</td>
<td>$\varepsilon_{1,p}(N) = \varepsilon_0 e^{-\left(\frac{\rho}{N}\right)^\beta}$</td>
<td>$\varepsilon_0$: Max. permanent strain at large number of cycles ($N$); $\rho$, $\beta$: Shape parameters</td>
</tr>
</tbody>
</table>

The last model can be used to predict permanent deformation in aggregate layers of thickness $h$:

$$\delta_{1,p} = \left(\frac{\varepsilon_0}{\varepsilon_r}\right) e^{-\left(\frac{\rho}{N}\right)^\beta} \varepsilon_v h$$

Where $\varepsilon_r$ and $\varepsilon_v$ are the resilient strain measured in the lab and the vertical strain obtained from layered elastic analysis.
Permanent Deformation Models

Uzan (1999)

- For Granular Materials:

\[
\log \frac{\varepsilon_p}{\varepsilon_r} = \left[ a_0 + a_1 \left( \frac{\theta + k_6}{Pa} \right) + a_2 \left( \frac{\tau_{oct}}{Pa} \right) \right] + \left[ b_0 + b_1 \left( \frac{\theta + k_6}{Pa} \right) + b_2 \left( \frac{\tau_{oct}}{Pa} \right) \right] \cdot \log N
\]

- For Plastic Subgrade:

\[
\log \frac{\varepsilon_p}{\varepsilon_r} = a_0 + b_0 \log N
\]
New MEPDG Rutting Model

\[ \delta_a (N) = \beta_{s1} \varepsilon_v h \left( \frac{\varepsilon_o}{\varepsilon_r} \right)^\beta \left[ e^{-\left(\frac{\rho}{N}\right)^\beta} \right] \]

- \( \delta_a \) = permanent deformation for the layer
- \( N \) = number of load repetitions
- \( \varepsilon_v \) = average predicted field vertical resilient strain
- \( h \) = thickness of the layer
- \( \varepsilon_o, \beta, \rho \) = lab material properties = \( f(M_R, w, \sigma_d, \sigma_\theta) \)
- \( \varepsilon_r \) = lab. resilient strain
- \( \beta_{s1} \) = field calibration factor
Synergistic Effect of Asphalt Modulus and Base Layer Modulus on Plastic Strains (VESYS Model) - (Ashtiani, 2007)
Stability Control
Safety Factor Method

- The rationale behind the safety factor method is to define a limit for the ratio of a measure of shear stresses, such as deviatoric stress to a measure of materials strength, such as unconfined compressive strength in geomaterials.
- Safety factor essentially defines a limiting threshold for the ratio of shear stresses induced by wheel loads and shear strength of the material determined in the lab.

\[
Safety \ Factor = \frac{Shear \ Strength}{Shear \ Stress}
\]

- This limit is believed to define a boundary below which the stability of the system is assured.
Stability of Aggregate Systems

• Shear Strength Ratio Concept

\[ \theta = \pi/4 + \phi/2 \]

\[ \tau_{\text{max}} = c + \sigma_n \tan \phi \]

\[ SSR = \frac{\sigma_d}{UCCS} \]

Shear Strength Ratio = \( \frac{\text{Shear Stress}}{\text{Shear Strength}} = \frac{\tau_f}{\tau_{\text{max}}} \)
Performance of the systems were assessed by shear strength criteria:

\[ SSR = \frac{\tau_{oct}}{UCCS} \]

- High fines content system with 2 percent stabilizer outperformed the control system at wet conditions.
Acceptability Criteria for Lightly Stabilized Aggregate Systems
Objective

Study of the synergistic effect of increasing fine content, moisture conditioning and stabilizer content on the performance of aggregate systems.
Analysis of the Level of Anisotropy

**Untreated Systems:**
- Reduction in the level of anisotropy with increasing fines content at optimum moisture state, the trend reverses at wet conditions.

**Stabilized Systems:**
- Reduction in the level of anisotropy with increasing fines content and stabilizer content in both optimum and wet moisture states.
- In case of 1% stabilizer, wet systems resulted in less anisotropic systems.
Measures of Nonlinearity

**Method One:**

\[ M_n = \frac{\varepsilon_f}{\varepsilon_r} \]

\( \varepsilon_r \): Resilient strain
  (maximum axial strain of the linear part of stress-strain curve)

\( \varepsilon_f \): Strain at failure

**Method Two:**

\[ M_n = \frac{q_f}{q_r} \]

\( q_r \): Deviatoric stress
  (maximum deviatoric stress of the linear part of stress-strain curve)

\( q_f \): Deviatoric stress at failure
Analysis of the Degree of Nonlinearity

- In untreated systems, degree of nonlinearity drastically increased with increasing fines content in the mix.
- In stabilized systems, degree of nonlinearity reduced with increase in stabilizer content.
Unconfined Compressive Strength

Stabilizer content %

V1, V2, V3, V4

UCCS (KPa) vs. Stabilizer content %
- Stabilized aggregate systems show less nonlinearity compared to their unstabilized counterparts. They tend to taper off fast and reach their asymptotic value.

- Less plastic deformation with increasing stabilizer content.
Permanent Deformation Results for 2% Stabilized Systems

- 2 percent cement stabilized systems with high fines content ($V_3$-2% and $V_4$-2%) tested at optimum moisture content performed better in terms of lower plastic strains compared to stabilized controlled system ($V_1$-2%).

- At wet conditions, excess fines content in gradation $V_4$ resulted in degradation of the performance. This suggests there is an optimum value for stabilizer and fines content exists that results in lower plastic deformation in the repeated load test.

- Gradation $V_3$-2% performed best in terms of the lowest plastic strain tested at wet conditions.
Permanent Deformation Results

Wet of Optimum Moisture State

- Microstrain after 10,000 cycles
- Stabilizer Content %

Optimum Moisture State

- Microstrain after 10,000 cycles
- Stabilizer Content %

V3 and V4 Failed
Analysis of Critical Strains

- In untreated systems, increase in fines content resulted in higher strains therefore smaller life span of the pavement.
- Gradation V3 with 2% stabilizer resulted in the lowest critical strains.
- There is a tipping point beyond which the increase in fines content and stabilizer results in degradation of the pavement performance.
Case Studies

• **Case 1:** Impact of Stabilizers (low levels)
• **Case 2:** Impact of subgrade treatment
• **Case 3:** Impact of asphalt thickness
Case Study

• **Case 1:** Impact of Stabilizers
• **Case 2:** Impact of subgrade treatment
• **Case 3:** Impact of asphalt thickness
<table>
<thead>
<tr>
<th>Tire Pressure: 100 psi</th>
<th>Contact Radius: 5.5 in</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>6”</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_y = 400,000$ psi</td>
<td></td>
</tr>
<tr>
<td>$\nu = 0.35$</td>
<td><strong>Linear Isotropic</strong></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>12”</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$k_1,$ $k_2$ and $k_3$</td>
<td></td>
</tr>
<tr>
<td>$E_x/E_y$</td>
<td><strong>Non-linear Anisotropic</strong></td>
</tr>
<tr>
<td>$G_{xy}/E_y$</td>
<td></td>
</tr>
<tr>
<td>$V_{xx}/V_{xy}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_y = 6000$ psi</td>
<td><strong>Linear Isotropic</strong></td>
</tr>
<tr>
<td>$\nu = 0.45$</td>
<td></td>
</tr>
</tbody>
</table>
Shear Strength Ratio for Control System

(Uniform gradation ASTM D-2940 without stabilizer)

<table>
<thead>
<tr>
<th>Radial Distance from Load Centerline</th>
<th>Shear Strength Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td>0.18</td>
<td>0.18</td>
</tr>
<tr>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>0.26</td>
<td>0.26</td>
</tr>
<tr>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>0.43</td>
<td>0.43</td>
</tr>
</tbody>
</table>
Shear Strength Ratio
Lightly Lime stabilized system

Radial Distance 25” from load centerline

12” Limestone Base
Shear Stress Distribution for Control System
(Uniform gradation ASTM – 2940 without stabilizer)

12" Limestone Base

Radial Distance 25 " from load centerline

17.3 psi
Shear Stress Distributions
High Fines Content System at Wet Conditions

24.6 psi

12” Limestone Base

Radial Distance 25” from load centerline
Shear Stress Distributions
Lightly Cement Stabilized High Fines System

8.2 psi

12” Limestone Base

Radial Distance 25” from load centerline
Case Study

- *Case 1:* Impact of Stabilizers
- **Case 2:** Impact of subgrade treatment
- *Case 3:* Impact of asphalt thickness
<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus Type</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asphalt</td>
<td>Linear Isotropic</td>
<td>$E_{AC}=400,000$ psi, $v=0.35$</td>
</tr>
<tr>
<td>UAB</td>
<td>Nonlinear Anisotropic</td>
<td>$Modular Ratio=0.15$</td>
</tr>
<tr>
<td>Treated Subgrade</td>
<td>Nonlinear Isotropic</td>
<td>$E_{CTB}=30,000$ psi, $v=0.3$</td>
</tr>
<tr>
<td>Natural Subgrade</td>
<td>Linear Isotropic</td>
<td>$E_{SG}=6,000$ psi, $v=0.45$</td>
</tr>
</tbody>
</table>
Impact of Subgrade Treatment on Bulk Stress in Base Layer

Bulk Stress (psi)

0.00 5.00 10.00 15.00 20.00 25.00 30.00 35.00 40.00

Z (in)

0 2 4 6 8 10 12 14 16 18

w/ Natural Subgrade  w/ Treated Subgrade
Impact of Subgrade Treatment on Octahedral Shear Stress in Base Layer

Octahedral Shear Stress (psi)

Z (in)

w/ Natural Subgrade  w/ Treated Subgrade
Impact of Subgrade Treatment on Vertical Modulus in Base Layer
Impact of Subgrade Treatment on Horizontal Modulus in Base Layer

![Graph showing the impact of subgrade treatment on horizontal modulus. The x-axis represents Horizontal Modulus (psi) ranging from 0 to 3000. The y-axis represents a range from 0 to 18. Two lines are plotted: one for 'W/ Natural Subgrade' and another for 'W/ Treated Subgrade.' The 'W/ Treated Subgrade' line starts lower and slopes upwards to the right, while the 'W/ Natural Subgrade' line starts higher and slopes upwards more steeply to the right.]
Impact of Subgrade Treatment on Pavement Response

0R-NAB

Percent Increase (%)

Bulk Stress  Oct. Shear Stress  Vertical Modulus  Horizontal Modulus

Top of Base  Middle of Base  Bottom of Base
Case Study

- **Case 1**: Impact of Stabilizers
- **Case 2**: Impact of subgrade treatment
- **Case 3**: Impact of asphalt thickness
<table>
<thead>
<tr>
<th>Depth</th>
<th>Material Properties</th>
<th>Modulus of Elasticity (psi)</th>
<th>Poisson's Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2” &amp; 4”</td>
<td>Linear Isotropic</td>
<td>( E_y = 400,000 )</td>
<td>( v = 0.35 )</td>
</tr>
<tr>
<td>12”</td>
<td>Non-linear Anisotropic</td>
<td>( k_1, k_2, k_3 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( E_x/E_y )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( G_{xy}/E_y )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( V_{xx}/V_{xy} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Linear Isotropic</td>
<td>( E_y = 10000 )</td>
<td>( v = 0.45 )</td>
</tr>
</tbody>
</table>

Tire Pressure: 100 psi
Contact Radius: 5.5 in

Stiff Subgrade

2” & 4”: \( E_y = 400,000 \) psi, \( v = 0.35 \)

12”:

\( k_1, k_2, k_3 \) and Modulus of Elasticity

\( E_x/E_y \), \( G_{xy}/E_y \), \( V_{xx}/V_{xy} \)
Distribution of Bulk Stress for two Aggregate Base Systems (Ashtiani, 2006)

<table>
<thead>
<tr>
<th></th>
<th>Top of Base</th>
<th>Top of Subgrade</th>
</tr>
</thead>
<tbody>
<tr>
<td>4&quot; AC with stiff Subgrade</td>
<td>38</td>
<td>8</td>
</tr>
<tr>
<td>2&quot; AC with Soft Subgrade</td>
<td>67</td>
<td>14</td>
</tr>
</tbody>
</table>

Bulk Stress (psi)

Bulk Stress (Psi)
Distribution of Octahedral Shear Stresses for Two Aggregate Base Systems (Ashtiani, 2006)

<table>
<thead>
<tr>
<th>Octahedral Shear Stress (psi)</th>
<th>Top of Base</th>
<th>Top of Subgrade</th>
</tr>
</thead>
<tbody>
<tr>
<td>4'' AC with stiff Subgrade</td>
<td>11</td>
<td>4</td>
</tr>
<tr>
<td>2'' AC with Soft Subgrade</td>
<td>34</td>
<td>8</td>
</tr>
</tbody>
</table>
Sensitivity Analysis of the ANN Model

- Overall changes in the output of the system when the feature of the database increases by one unit.

\[ \delta_k = -\frac{\partial J}{\partial t_k} \]

- Where: \( \delta_k \) is the sensitivity of the model to feature \( k \),

\( t_k \) is the units’ net activation
Choice of Material Model (ICAR Protocol)

✓ Resilient Modulus in Vertical Direction

\[ E_y = K_1 \left( \frac{\theta}{P_a} \right)^{K_2} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{K_3} \]

✓ Resilient Modulus in Horizontal Direction

\[ E_x = K_4 \left( \frac{\theta}{P_a} \right)^{K_5} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{K_6} \]

✓ Shear Modulus in xy Plane

\[ G_{xy} = K_7 \left( \frac{\theta}{P_a} \right)^{K_8} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{K_9} \]
More Angular Particles  →  Better Aggregate Interlock  →  Less Anisotropic Behavior
More Cubical particles  \( \rightarrow \) Less Anisotropic Behavior

More Flat and Elongated particles  \( \rightarrow \) More Anisotropic Behavior
More Roughly Textured Particles  Less Anisotropic Behavior
Gradations with larger particles lead to less anisotropic behavior.
Problem 1.

Perform a sensitivity analysis of the modular ratios of two consecutive layers (e.g. $E_{AC}/E_{Base}$) in a three layer system. Provide plots of vertical and shear stress distributions vs. depth for the following scenarios and discuss your findings.

- Assume the modulus of the base layer as your last 5 digits of university ID number and develop plots of vertical and shear stress distributions vs. pavement depth along the load centerline for $E_{AC}/E_{Base} = 5, 10, 20, 30, 40, 50, 100$.
- Assume the thickness of the surface layer as 4 inches, and the thickness of the second layer (base layer) as 12 inches.
- Consider a soft subgrade with modulus of 4,000 psi and a stiff subgrade of 12,000 psi for your simulations.
- Assume a tire pressure of 100 psi acting on the surface of the pavement; assume the tire contact radius of 5.5 inches.
- Assume any other parameter/material property that you might need for your simulations; provide a justification for your assumptions.
Problem 2.

A proposed pavement design under construction for a highway consists of 6 inches of Hot Mix Asphalt (HMA) layer with modulus of 500,000 psi, and 10 inches of unbound granular base (UAB) layer over a relatively soft subgrade with modulus of 5,000 psi.

Determine the number of 18,000 lbs. ESALs that this pavement structure can support to reach rutting and fatigue failure. Use both Asphalt Institute (AI) and Shell transfer functions and discuss your results.

a) Vary the modulus of the base layer from 15,000 psi to 60,000 psi in 15,000 psi increments.

b) Assume proper Poisson ratio for each layer when you analyze the pavement system in a layered elastic software (such as WINJULIA or KENLAYER).

c) Assume the contact radius for the tire footprint on the pavement as 5.5 inches.

d) Discuss the influence of base layer modulus on the critical pavement responses.
$I_1 = 275.6 \text{ kPa}$

Granite: $y = 0.0081x^{1.6855}$  
$R^2 = 0.9982$

Limestone: $y = 0.01x^{1.5533}$  
$R^2 = 0.9715$

(a) Correlations of $J_2$ with Accumulated Plastic Strain at 10,000 Load Cycles
SQRT(J₂) = 111.4 kPa

Granite: y = 27025x⁻¹.838
R² = 0.9924

Limestone: y = 79537x⁻².04
R² = 0.9850

(b) Correlations of I₁ with Accumulated Plastic Strain at 10,000 Load Cycles