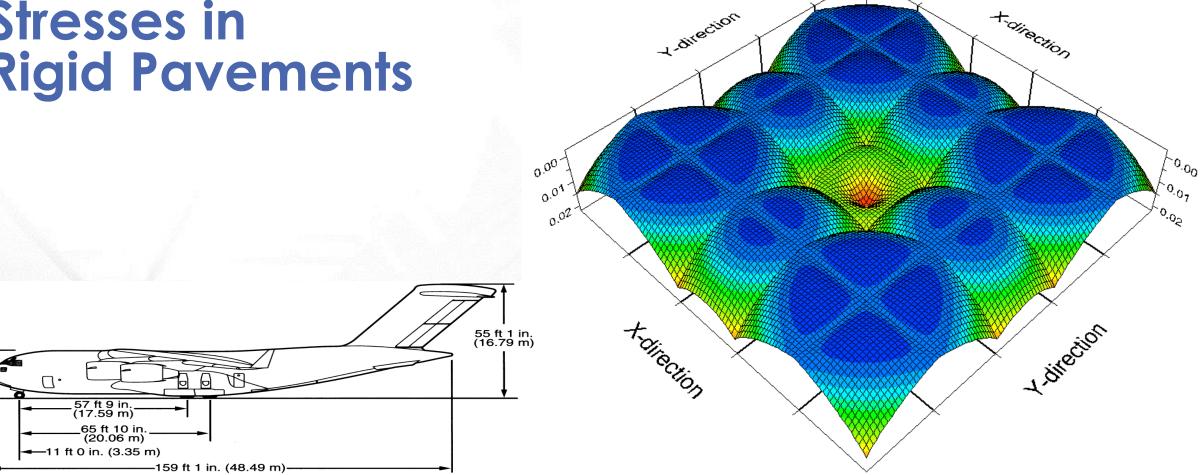
### Lecture 3:

### Stresses in **Rigid Pavements**



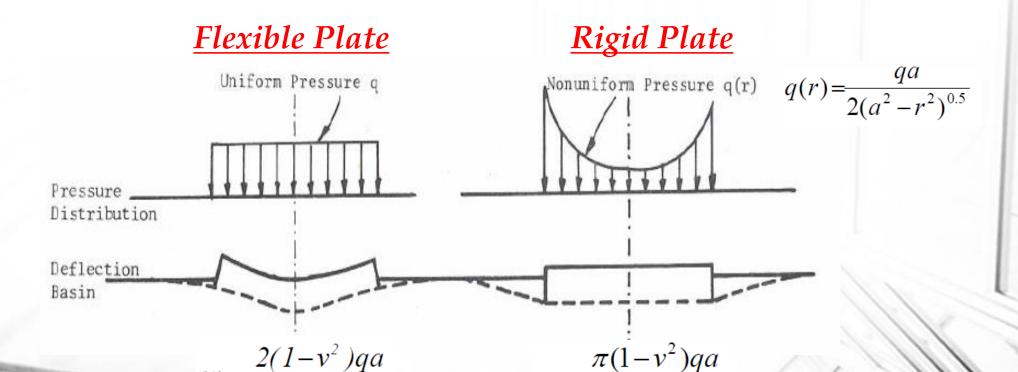
# Nature of Responses under Flexible and Rigid Plates

#### *Flexible plate:*

- ✓ Uniform Contact Pressure
- ✓ Variable Deflection Profile

#### Rigid Plate plate:

- ✓ Non-Uniform Contact Pressure
- ✓ Equal Deflection



## Comparison of Deflections at the Surface Rigid vs. Flexible Plate

Flexible Plate
$$d_o = \frac{2\sigma_o a (1-\mu^2)}{E}$$

Rígíd Plate
$$d_o = \frac{\pi \sigma_o a \left(1 - \mu^2\right)}{2E}$$

$$\frac{d_{o}^{\text{flexible}}}{d_{o}^{\text{rigid}}} = \frac{2 \frac{\sigma_{o} a \left(1 - \mu^{2}\right)}{E}}{\frac{\pi}{2} \frac{\sigma_{o} a \left(1 - \mu^{2}\right)}{E}} = \frac{4}{\pi} = 1.27 \implies \frac{d_{o}^{\text{rigid}}}{d_{o}^{\text{flexible}}} = \frac{\pi}{4} = 0.79$$

The deflection under a rigid plate is 79% of that under a flexible plate.

# Contact Pressure and Deflection Calculation under the Rigid plate

➤ Ullidtz (1987) gave the distribution of contact pressure under a rigid plate as:

$$p(r) = \frac{pa}{2(a^2 - r^2)^{0.5}}$$

 $\Delta_o = \frac{\pi (1 - \mu^2) pa}{2F}$ 

- ➤ Notice that the distribution of the contact pressure is a function of radial offset from the load centerline.
- ➤ By integrating a point load over the contact area of the plate, the deflection can be calculated as:
- Assuming  $\mu$ =0.5, the surface deflection at centerline can be calculated as:

$$\Delta_o = \frac{1.18 \, pa}{E}$$

### Stresses in Rigid Pavements

#### A. Stresses due to Environment

- > Warping (or Curling) Stresses
  - \*Stresses due to temperature differential or change in humidity (temperature or moisture gradient)

#### Location:

- ✓ Interior Loading
- ✓ Edge loading
- √ Corner Loading
- > Shrinkage/Expansion Stresses

#### **B. Stresses due to External Loading**

> Such as stresses induced by traffic loads

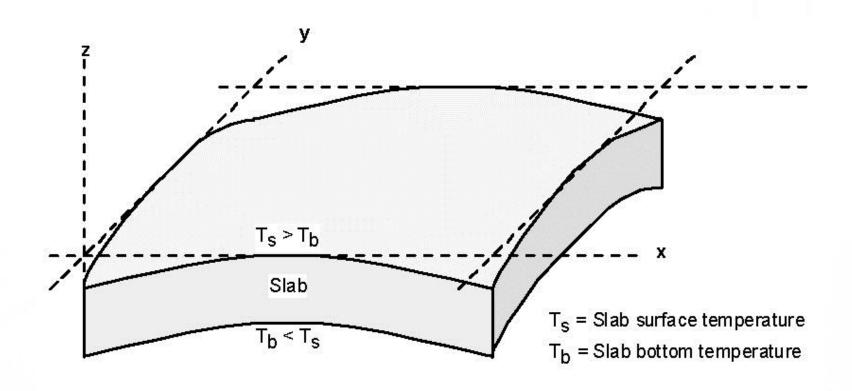
#### Location:

- ✓ Interior Loading (sufficiently away from the joints- no discontinuity effect)
- ✓ Edge loading
- ✓ Corner Loading

#### C. Other Stresses

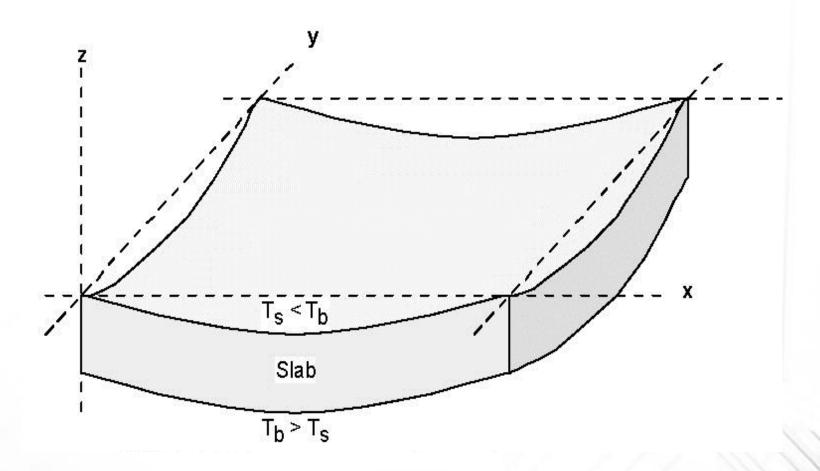
> Such as frictional forces between the slab and foundation

### Warping Stress - Day Time



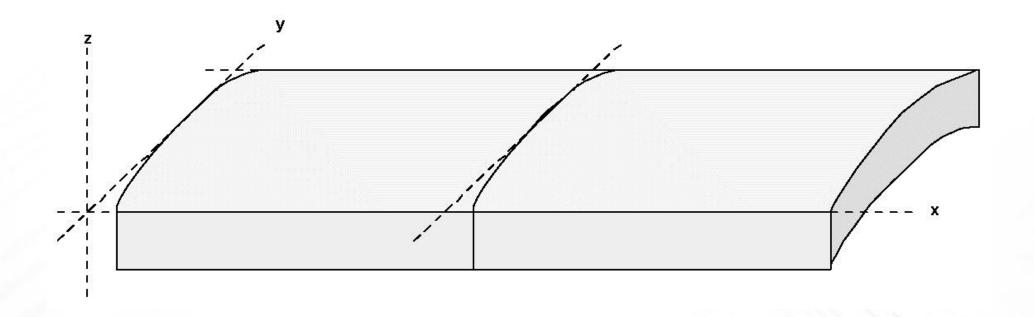
Slab Surface Temperature > Slab Bottom Temperature

### Warping Stress - Night Time



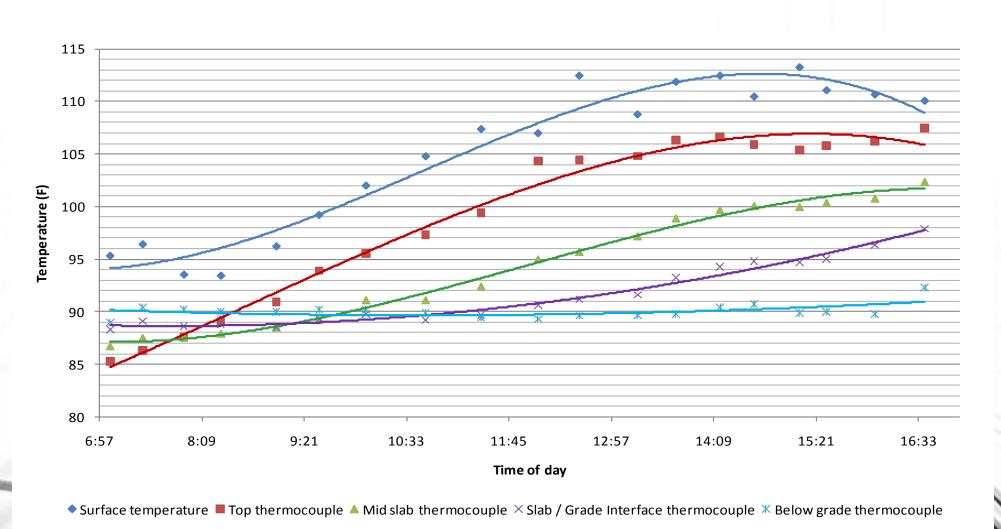
Slab Bottom Temperature > Slab Surface Temperature

### **Constrained Transverse Joints**



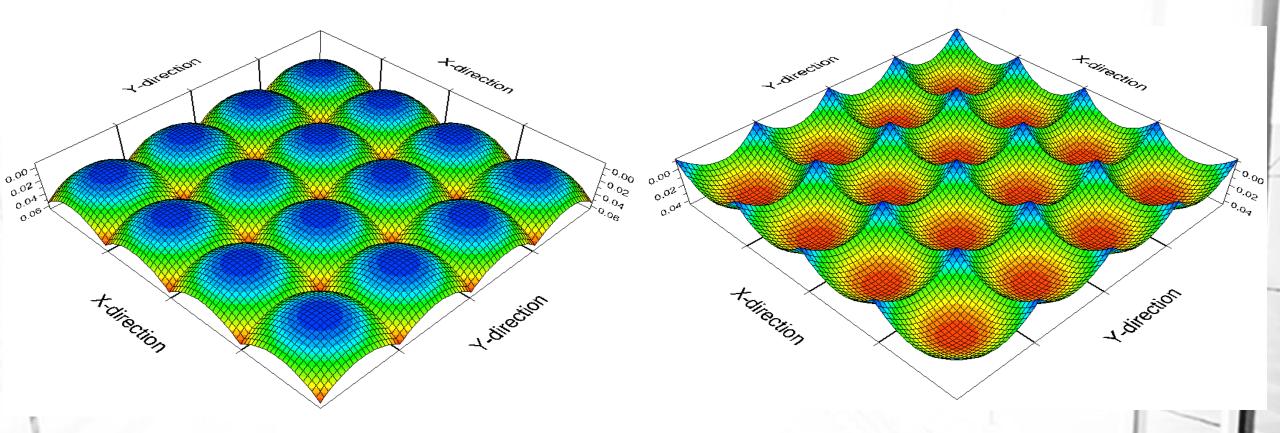
Slab Surface Temperature > Slab Bottom Temperature

#### Temperature Variation for Precast Concrete Panel Installed using HDP Deep Injection (Ashtiani, 2011)



#### **Typical Day Time Warping**

#### **Typical Night Time Warpingg**



Top of the PCC Layer is Warmer Bottom of the PCC Layer is Cooler Top of the PCC Layer is Cooler Bottom of the PCC Layer is Warmer

#### Calculation of Warping Stresses Edge Stress

$$\sigma_{t} = \frac{CEe\Delta T}{2}$$

 $\sigma_t$  = Slab edge warping stress (psi)

E = Modulus of elasticity of PCC (psi)

e = Thermal coefficient of PCC (approximately 0.000005 /F)

 $\Delta T$  = Temperature differential between the top and bottom of the slab (F)

C = Coefficient, function of slab length and the radius of relative stiffness, *l* 

#### Radius of Relative Stiffness Definition

$$l = \sqrt[4]{\frac{Eh^3}{12k(1-\upsilon^2)}}$$

v = Poisson Ratio

E = Modulus of Elasticity (psi)

*l* = Radius of Relative Stiffness (in)

k = Modulus of Subgrade Reaction (pci)

h = Slab Thickness (in)

# Westergaard's Model of Subgrade Reaction

➤ Elastic layered theory can't be applied for jointed rigid pavements due to the fact that one of the assumptions of the layered theory was that layers are infinitely long in horizontal direction (no effect of discontinuity at joints). To solve this problem Westergaard (1925) assumed that a rigid pavement could be considered as a slab on a Winkler foundation. In other words, the foundation reaction (or vertical stress) equals to the deflection times a constant (k) called modulus of subgrade reaction.

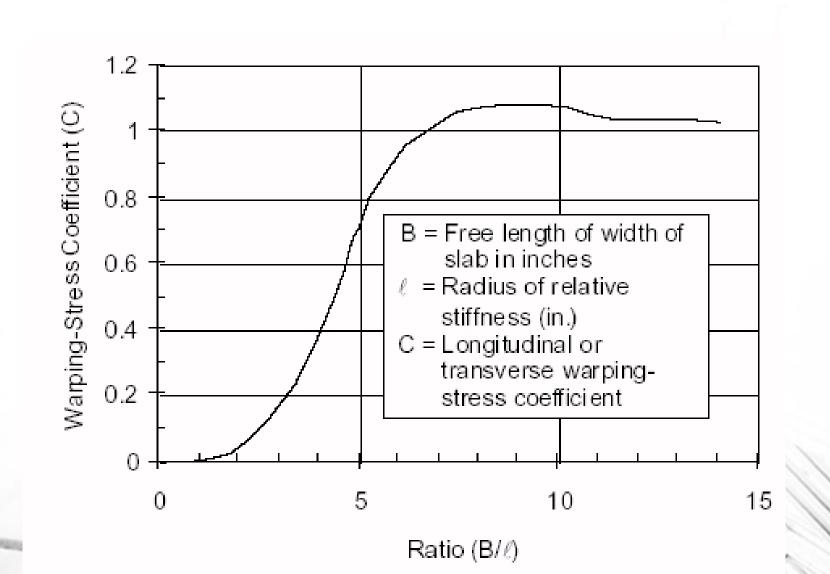
Note that the modulus of subgrade reaction (k) has unit of (pci).

PCC Slab

PCC Slab

PCC Slab

#### Determination of the Warping Stresses, Coefficient (C)



## Calculation of Warping Stresses, Cont. Interior Stress

$$\sigma_{t} = \frac{Ee\Delta T}{2} \left[ \frac{C_{1} + \upsilon C_{2}}{1 - \upsilon^{2}} \right]$$

 $\sigma_t$  = Slab interior warping stress (psi)

E = Modulus of elasticity of PCC (psi)

e = Thermal coefficient of PCC (~0.000005/F)

v = Poisson's ratio for PCC

 $C_1$  = Coefficient in direction of calculation

 $C_2$  = Coefficient in direction perpendicular to  $C_1$ 

## Calculation of Warping Stresses, Cont. Corner Stress

$$\sigma_t = \frac{Ee\Delta T}{3(1-\upsilon)}\sqrt{\frac{a}{l}}$$

 $\sigma_t$  = Corner Warping Stress (psi)

E = Modulus of Elasticity of PCC (psi)

e = Thermal Coefficient of PCC (~0.000005/F)

 $\Delta T$  = Temperature Differential between the Top and Bottom of the Slab (F)

v = Poisson's Ratio for PCC

a = Contact Radius for Corner Load

*l* = Radius of Relative Stiffness

# Stresses due to External Loads Westergaard Equations

➤ Interior loading (tensile stress at the bottom of the slab)

$$\sigma_{i} = \frac{0.3162(W)}{h^{2}} \left[ 4 \log_{10} \left( \frac{\ell}{b} \right) + 1.069 \right]$$

➤ Edge loading loading (tensile stress at the bottom of the slab)

$$\sigma_{e} = \frac{0.572(W)}{h^2} \left[ 4 \log_{10} \left( \frac{\ell}{b} \right) + 0.359 \right]$$

Corner loading (tensile stress on the top of the slab)

$$\sigma_{c} = \frac{3(W)}{h^{2}} \left[ 1 - \left( \frac{a\sqrt{2}}{\ell} \right)^{0.6} \right]$$

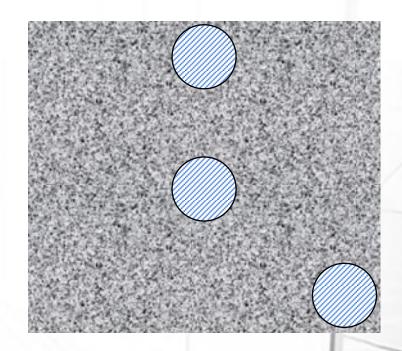
W = Wheel load (lb) h = Slab thickness (in.)

a = Radius of Wheel Contact Area (in.)

*l* = Radius of Relative Stiffness (in.)

b = Radius of Resisting Section (in.)

 $b = \sqrt{1.6(a^2) + h^2} - 0.675(h)$ 

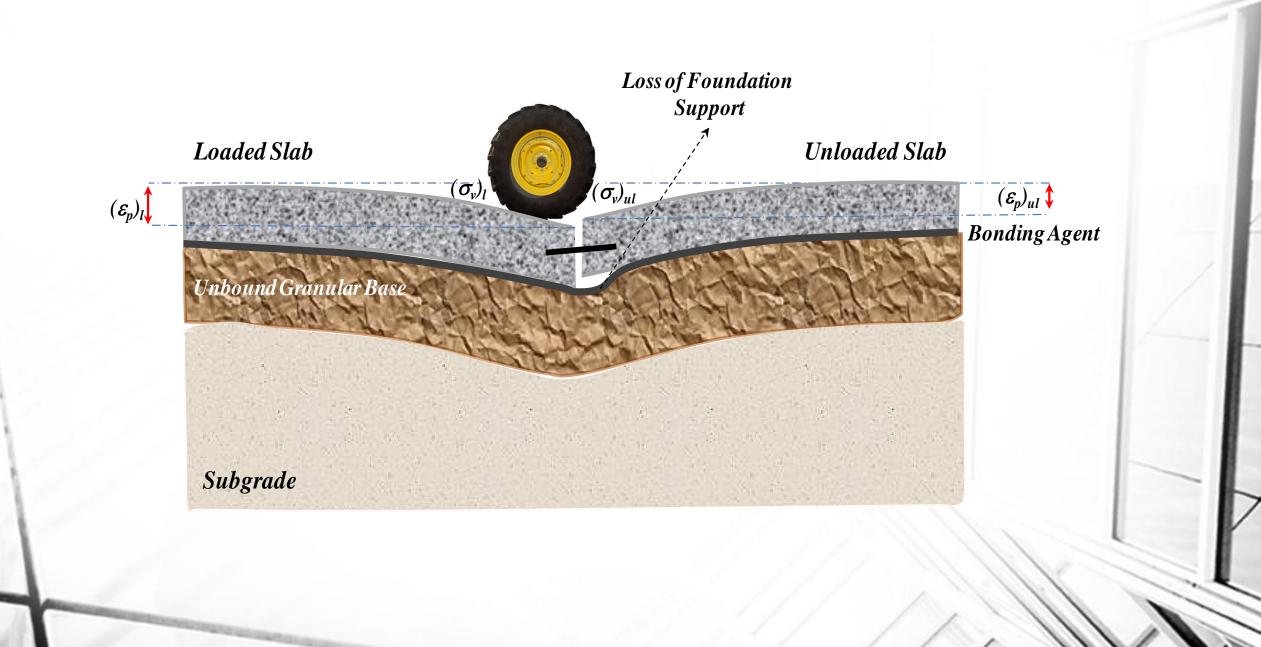


#### Slab Expansion/Contraction

$$z = CL[e\,\Delta t + \delta]$$

- z = Joint opening (or change in slab length, in.)
- C = Base/slab frictional restrain factor (0.65 for stabilized bases; 0.80 for granular bases)
- L = Slab length (in.)
- e = PCC coefficient of thermal expansion by aggregate type (e.g.,  $6.0 \times 10^{-6}$ /F for gravel;  $3.8 \times 10^{-6}$ /F for limestone)
- $\Delta t = Maximum temperature range$
- $\delta$  = Shrinkage coefficient of concrete (e.g., 0.00045 in./in. for indirect tensile strength of 500 psi)

## Load Transfer Efficiency (LTE) in Rigid Pavements



# Load Transfer Efficiency (LTE) in Rigid Pavements

- Load Transfer Efficiency (LTE)
  - ✓ Deflection based, LTE $_{\delta}$
  - $\checkmark$  Stress based, LTE $_{\sigma}$
  - √ FAA Criteria (stress based), LT

$$LT = \frac{Stress_{free\ edge} - Stress_{loaded\ slab}}{Stress_{free\ edge}}$$

$$= 1 - \frac{Stress_{loaded\ slab}}{Stress_{free\ edge}}$$

$$LTE_{\delta} = \frac{Deflection_{unloaded\ slab}}{Deflection_{loaded\ slab}}$$

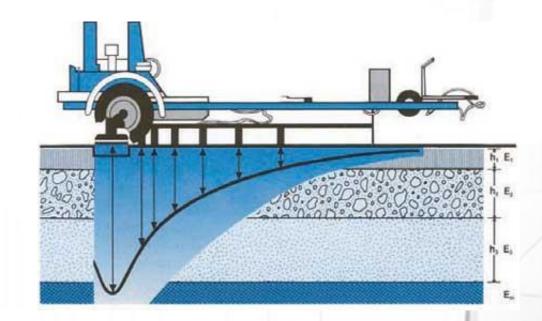
$$LTE_{\sigma} = \frac{Stress_{unloaded \, slab}}{Stress_{loaded \, slab}}$$

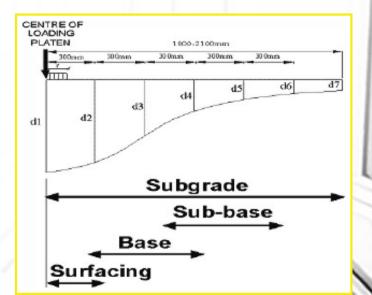
$$LT = \frac{LTE_{\sigma}}{1 + LTE_{\sigma}}$$

$$LTE_{\delta} = \frac{\left[1206\left(\frac{a}{l}\right) + 377\right]LTE_{\sigma}^{2} - 393\left(\frac{a}{l}\right)LTE_{\sigma}^{3}}{1 + 689\left(\frac{a}{l}\right)LTE_{\sigma} + \left[370 - 154\left(\frac{a}{l}\right)\right]LTE_{\sigma}^{2}}$$

### Falling Weight Deflectometer (FWD)

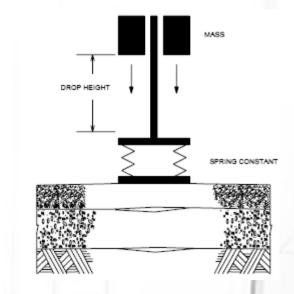
- ➤ A falling weight deflectometer (FWD) is a device designed to simulate deflection of a pavement surface caused by a fast-moving truck. The FWD generates a load pulse by dropping a weight.
- ➤ This load pulse is transmitted to the pavement through a 300-millimeter (mm) diameter circular load plate.
- ➤ The load pulse generated by the FWD momentarily deforms the pavement under the load plate into a dish or bowl shape.
- ➤ Based on the force imparted to the pavement and the shape of the deflection basin, it is possible to estimate the stiffness of the pavement layers by using various computational methods.





## Falling Weight Deflectometer (FWD) General Definitions

- ✓ Non-destructive test equipment for pavements.
- ✓ Imparts an impact load to a pavement structure.
- ✓ Measures deflection of the pavement surface at different radial offsets.
- ✓ Different types based on the application : Light Weight Deflectometer (LWD), Falling weight Deflectometer (FWD), and Heavy Weight Deflectometer (HWD).















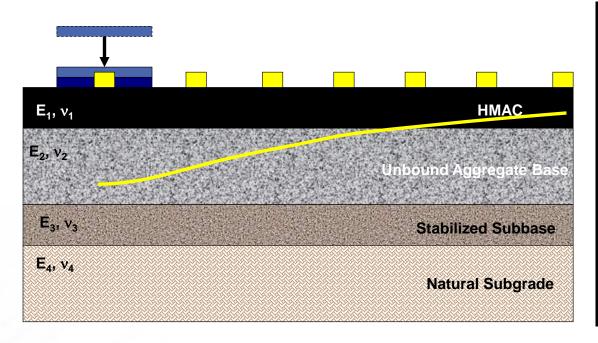
#### Components of FWD

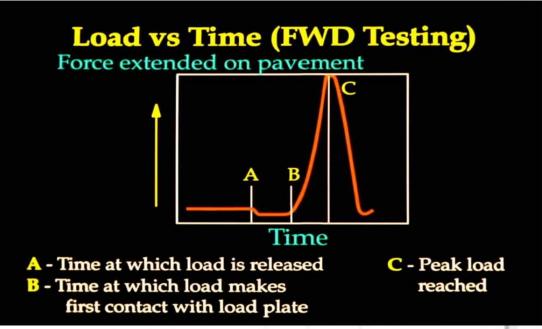
- ➤ Load Cell
- >LVDT, Geophones, Accelerometers
  - > Displacement measurement
- >Infrared temperature gages
  - ➤ Pavement Surface Temperature
  - ➤ Air Temperature
- > Electronic Distance Measurement
- >Control/Data Acquisition Unit





#### **FWD Details**



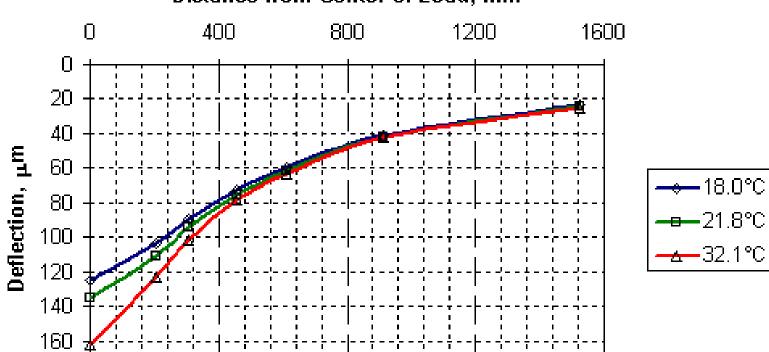


- > Impact Load created from dropping weights from specified height.
- > Load cell used to measure the impulse loading.
- > Heavy-duty load cell required to support in excess of 60 kips in magnitude.
- Deflection profile is key output.
- > Temperature and load data used with deflections to back-calculate pavement structure characteristics.

## Temperature Dependency of Deflection Profile in a Flexible Pavement

180

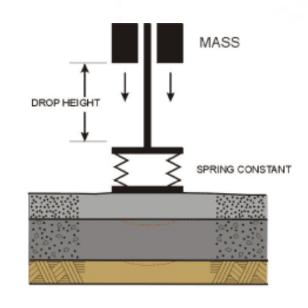


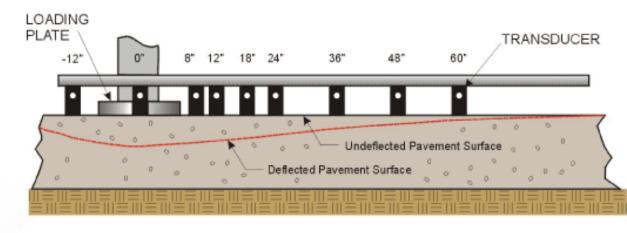


## Application of FWD/HWD in Pavement Engineering

- > Determination of the in-situ layer moduli.
- > Estimation of the structural capacity and analysis of the remaining life.
- > Determination of the load transfer efficiency of joints in concrete pavements.
- > Pavement management.



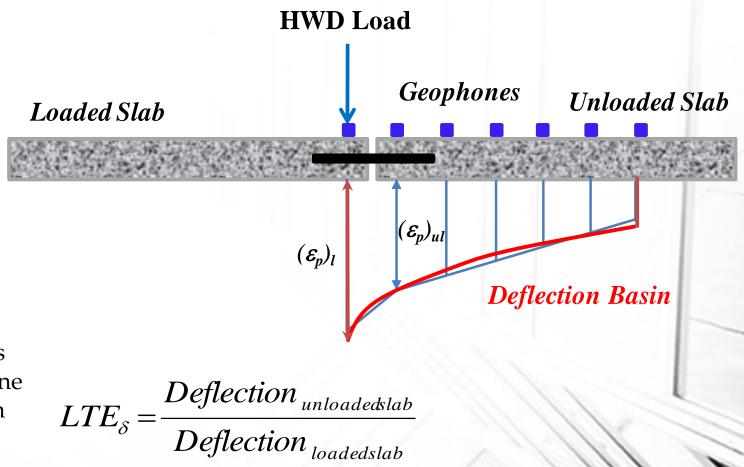




## Application of Falling Weight Deflectometer (FWD) for calculations of Load Transfer Efficiency (LTE)



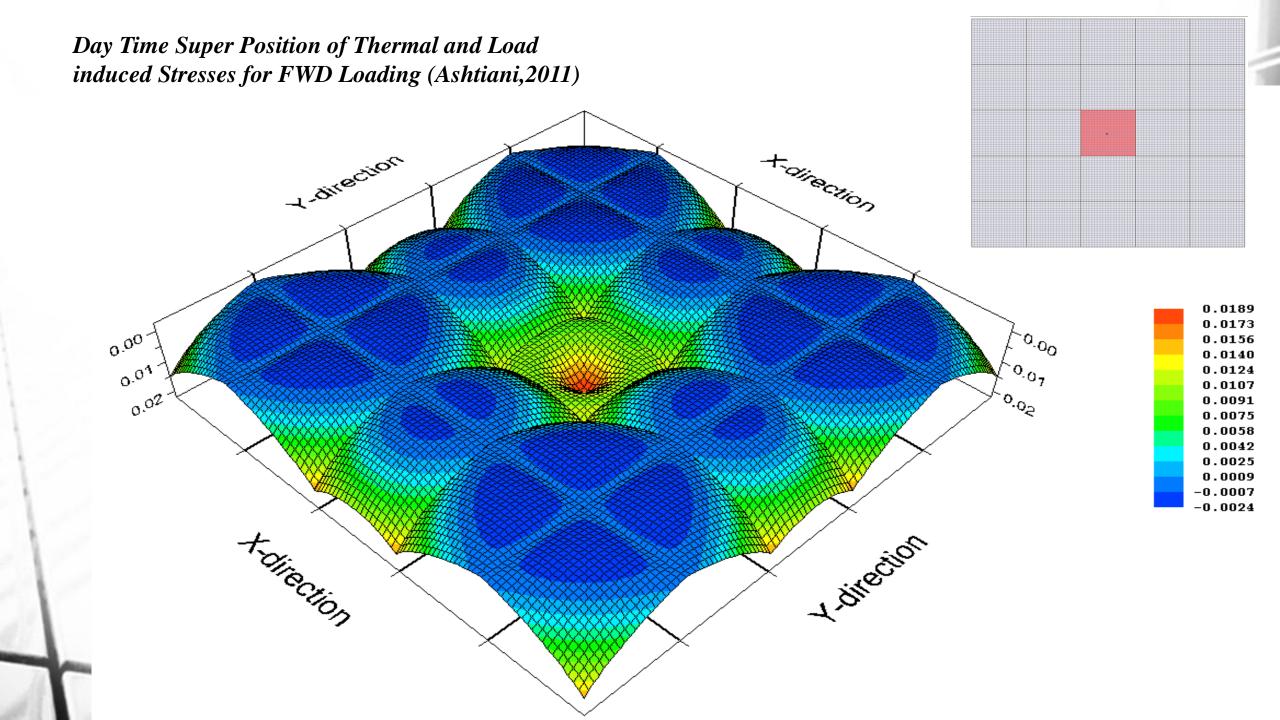
Deflection based LTE (ratio of plastic strains at the two sides of the joints) is the most common method to determine LTE and joint stiffness of PCC slabs in pavement industry.



### Case Study (I)

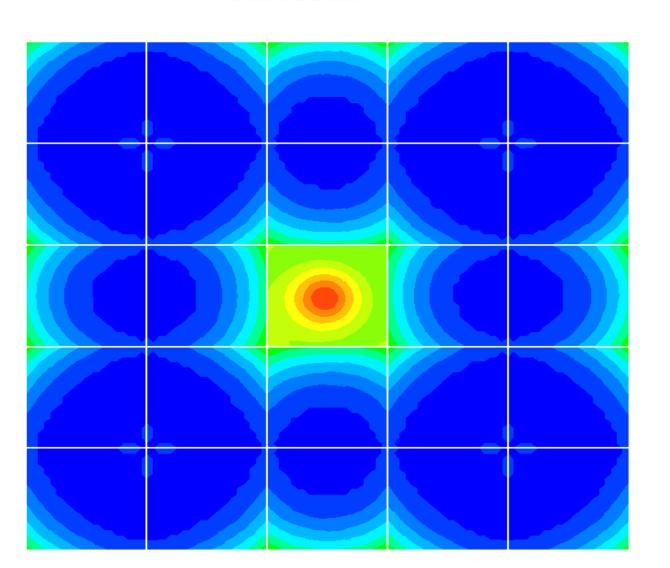
# Superposition of Thermal Stresses and HWD Load

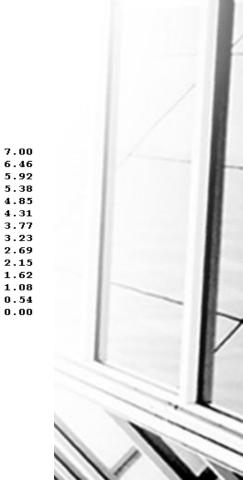
Rapid Damage Repair using Precast Concrete Slabs Tyndall AFB, (Ashtiani, 2010)

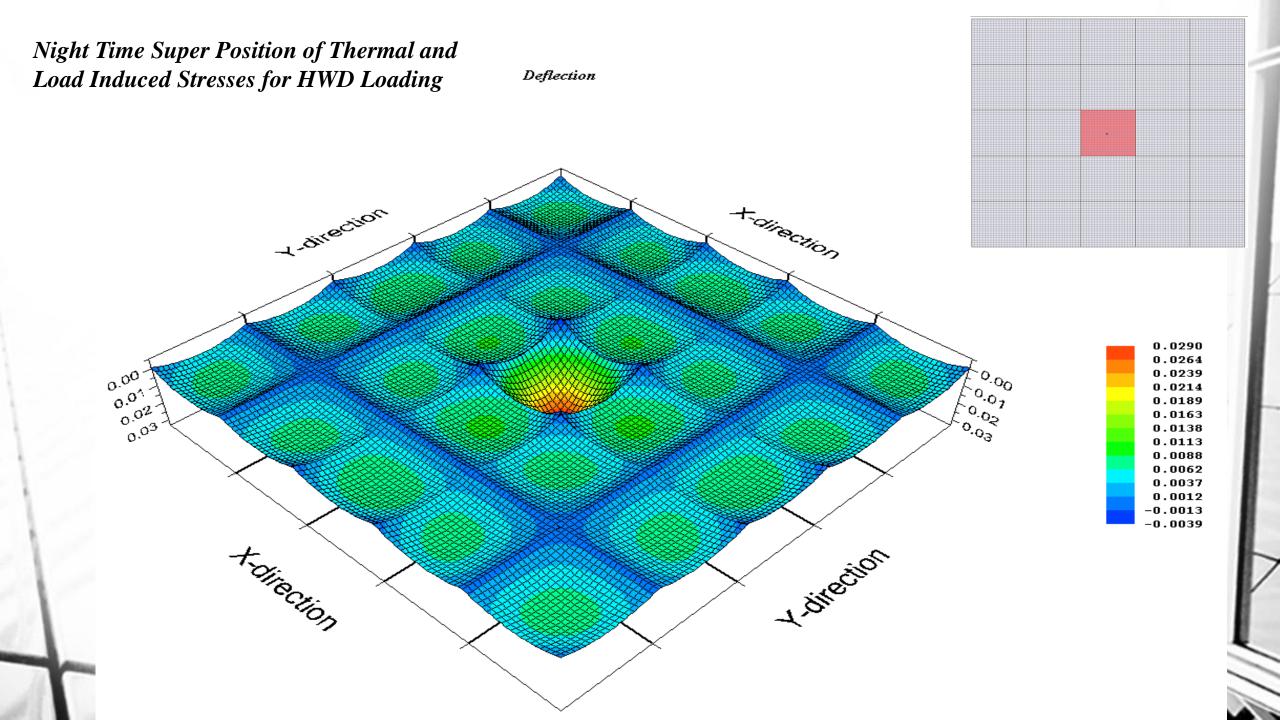


## Distribution of Vertical Stresses at the Top of the Subgrade (Day Time)

Stresses in Z-direction

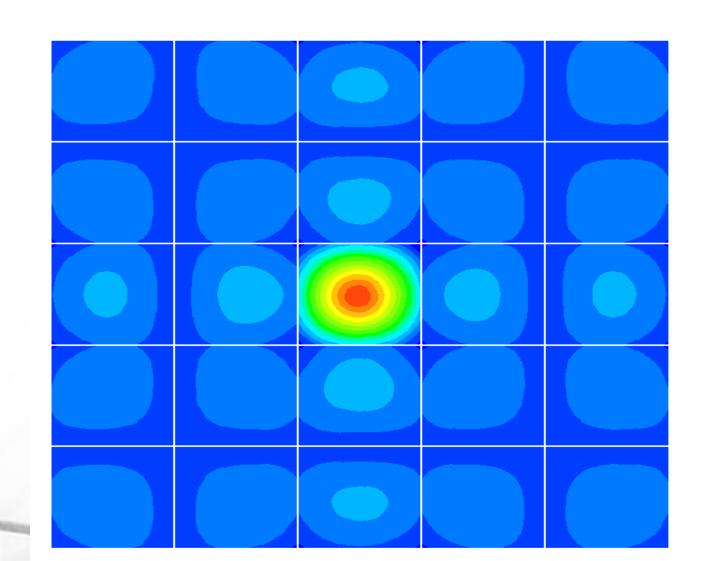


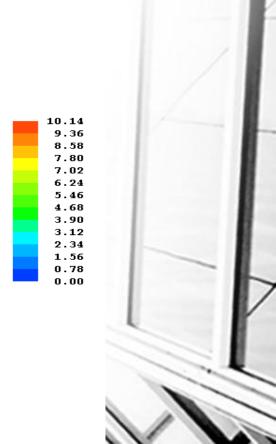




## Distribution of Vertical Stresses at the Top of the Subgrade (Night Time)

Stresses in Z-direction





### Case Study (II)

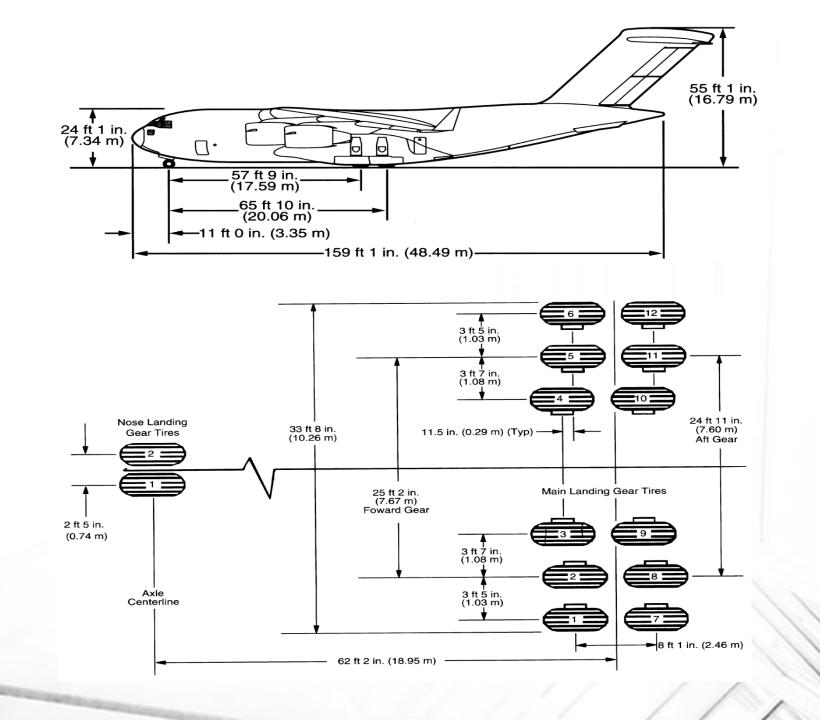
Superposition of Thermal Stresses and Stresses due to C17 Aircraft Landing Gear

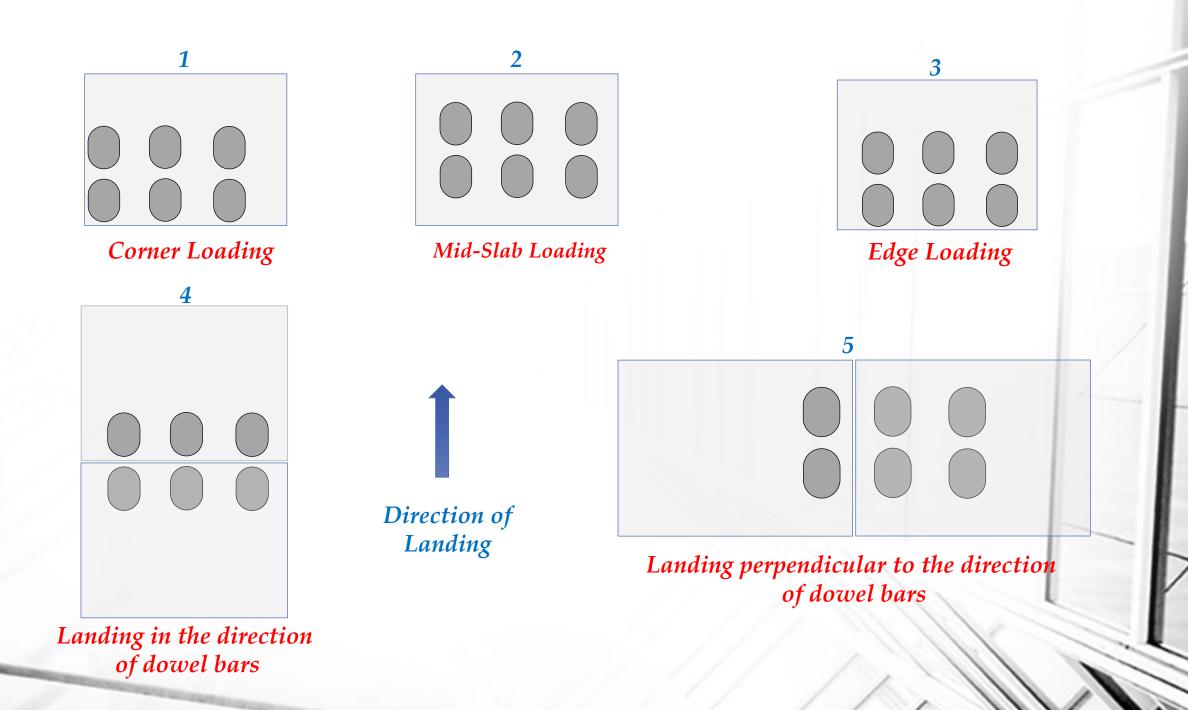
Rapid Damage Repair using Precast Concrete Slabs Tyndall AFB, (Ashtiani, 2010)

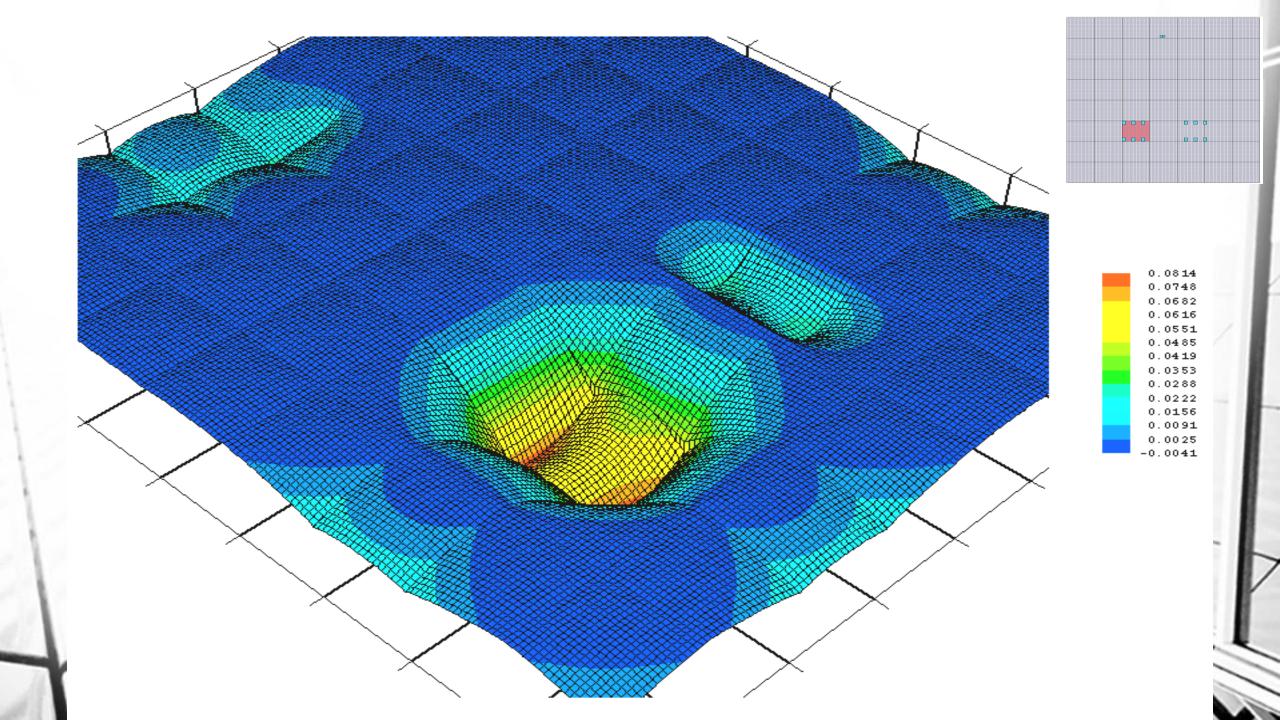




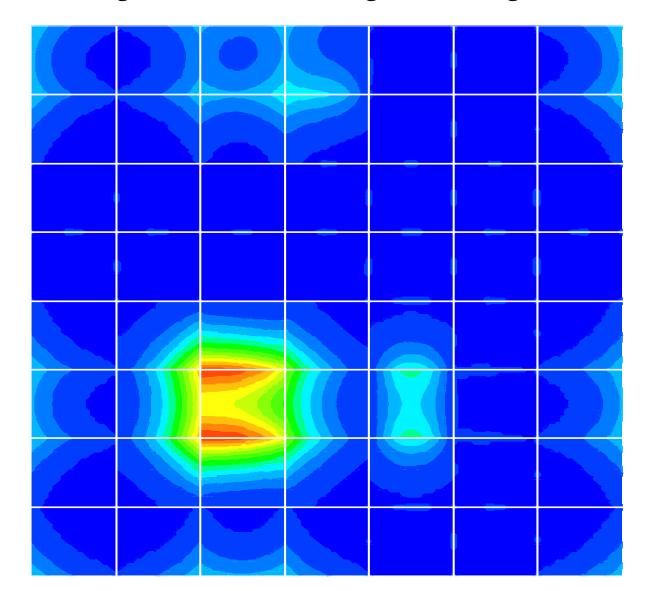


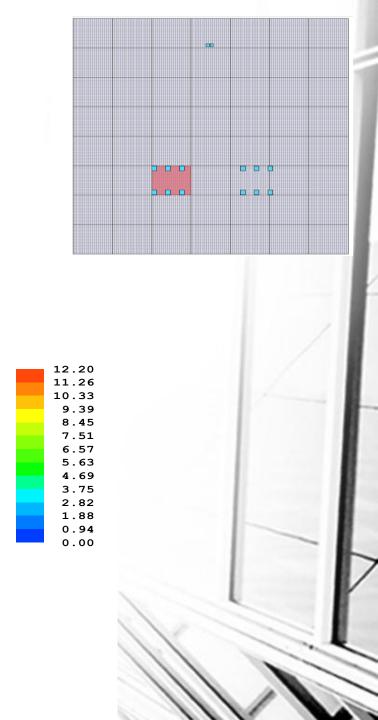






Distribution of Vertical Stresses at the Top of the Subgrade (Corner/Edge Loading for Slab #1)





# Sensitivity Analysis of Parameters of Different Measures of Load Transfer Efficiency (LTE)

Sensitivity Analysis of the parameters of the LTE model.

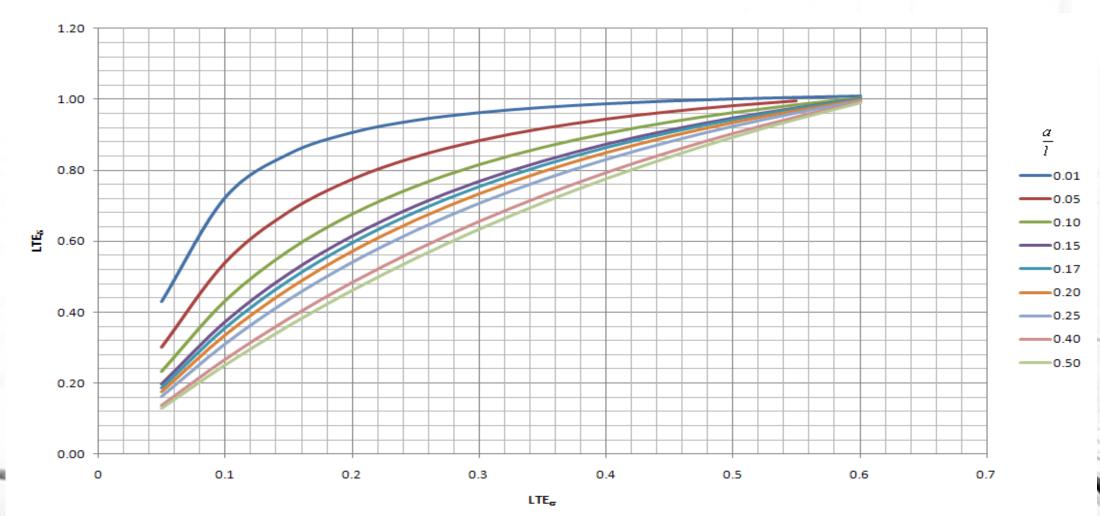
$$LTE_{\delta} = \frac{\left[1206\left(\frac{a}{l}\right) + 377\right]LTE_{\sigma}^{2} - 393\left(\frac{a}{l}\right)LTE_{\sigma}^{3}}{1 + 689\left(\frac{a}{l}\right)LTE_{\sigma} + \left[370 - 154\left(\frac{a}{l}\right)\right]LTE_{\sigma}^{2}}$$

E (psi)	1,000,000	1,500,000	2,000,000	2,500,000	3,000,000	3,500,000	4,000,000	4,500,000	5,000,000
1	25.82	28.57	30.71	32.47	33.98	35.32	36.51	37.61	38.61
a/I	0.23	0.21	0.20	0.18	0.18	0.17	0.16	0.16	0.16

						a/I				
		0.01	0.05	0.10	0.15	0.17	0.20	0.25	0.40	0.50
	0.05	0.43	0.30	0.23	0.20	0.19	0.18	0.16	0.14	0.13
	0.1	0.72	0.54	0.43	0.37	0.36	0.34	0.31	0.27	0.25
	0.15	0.85	0.68	0.57	0.51	0.49	0.47	0.44	0.38	0.36
	0.2	0.91	0.77	0.68	0.61	0.60	0.57	0.54	0.48	0.46
	0.25	0.94	0.84	0.75	0.70	0.68	0.66	0.63	0.57	0.55
1.75	0.3	0.96	0.88	0.82	0.77	0.75	0.73	0.71	0.66	0.63
LTE <sub>σ</sub>	0.35	0.98	0.92	0.86	0.83	0.81	0.80	0.77	0.73	0.71
	0.4	0.99	0.94	0.90	0.87	0.86	0.85	0.83	0.79	0.78
	0.45	1.00	0.97	0.94	0.91	0.91	0.90	0.88	0.85	0.84
	0.5	1.00	0.98	0.96	0.95	0.94	0.94	0.93	0.90	0.89
	0.55	1.01	1.00	0.99	0.98	0.97	0.97	0.96	0.95	0.95
	0.6	1.01	1.01	1.00	1.00	1.00	1.00	1.00	0.99	0.99

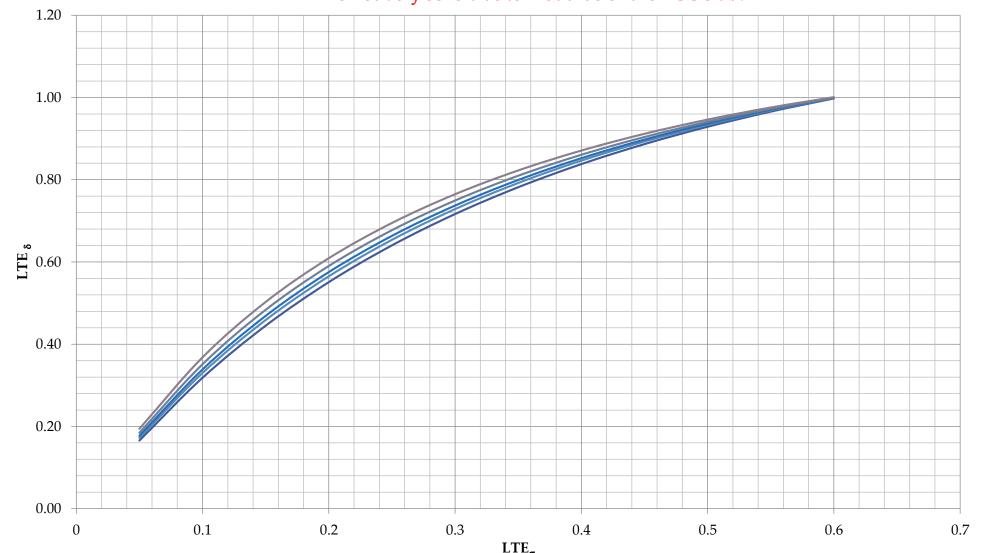
### Sensitivity of the Load Transfer Efficiency to the Parameters of the Model, effect of (a/l)

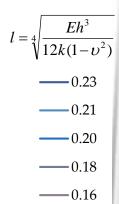
$$LTE_{\delta} = \frac{\left[1206\left(\frac{a}{l}\right) + 377\right]LTE_{\sigma}^{2} - 393\left(\frac{a}{l}\right)LTE_{\sigma}^{3}}{1 + 689\left(\frac{a}{l}\right)LTE_{\sigma} + \left[370 - 154\left(\frac{a}{l}\right)\right]LTE_{\sigma}^{2}}$$

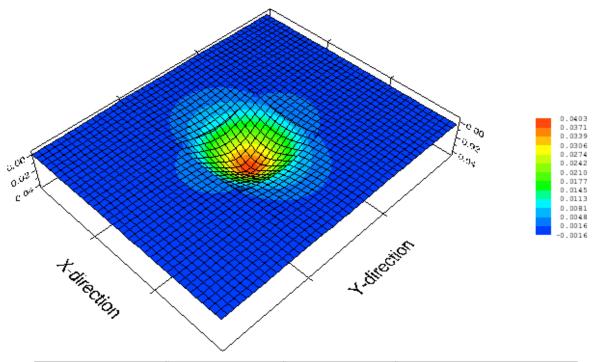


E (psi)	1,000,000	1,500,000	2,000,000	2,500,000	3,000,000	3,500,000	4,000,000	4,500,000	5,000,000
I	25.82	28.57	30.71	32.47	33.98	35.32	36.51	37.61	38.61
a/I	0.23	0.21	0.20	0.18	0.18	0.17	0.16	0.16	0.16

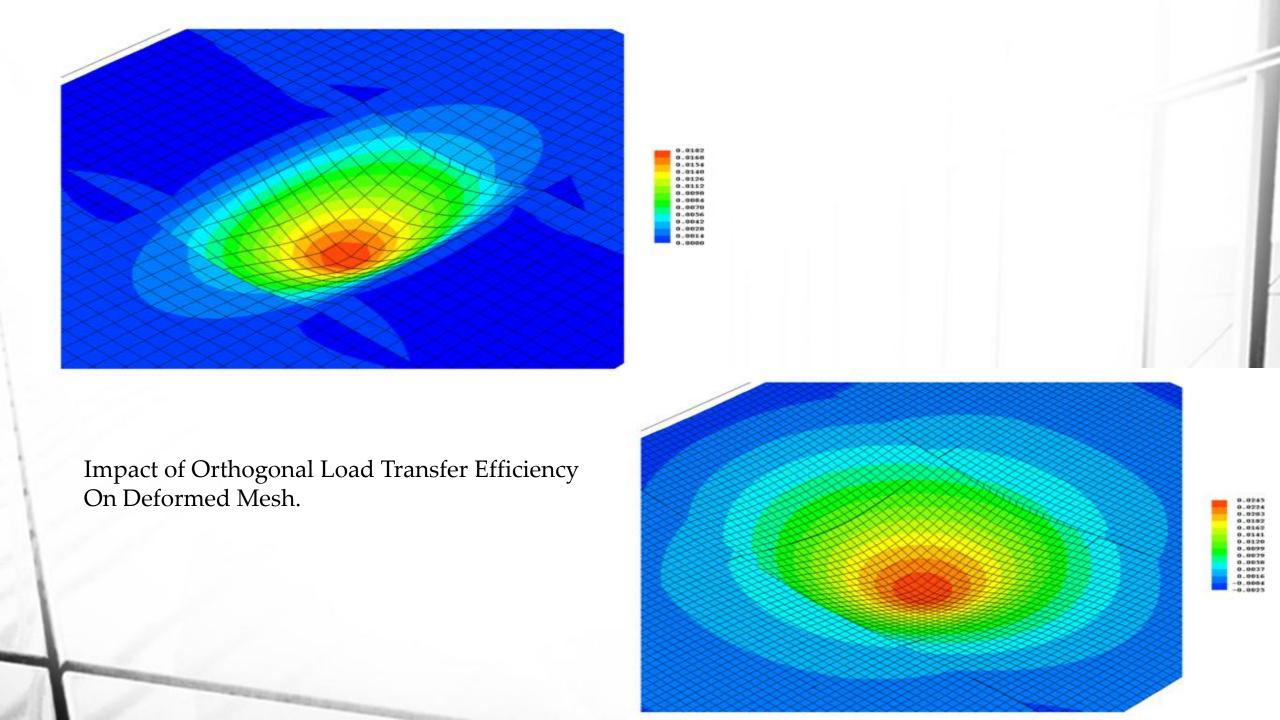
For plate loading with r=6 in k=100 pci and h=9 in, LTE is not very sensitive to modulus of the PCC slab.





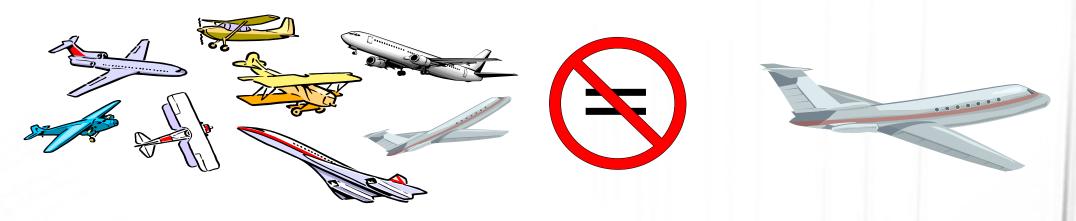


Simulation ID	$LTE_{x}\left(\%\right)$	$E_{pcc}$ (ksi)	FE Calculated $arepsilon_p$
1-90-1	90	1000	0.0407
2-90-1	90	2000	0.0336
<i>3-90-1</i>	90	3000	0.0381
<i>4-90-1</i>	90	4000	0.0259
<i>5-90-1</i>	90	5000	0.0249
1-95-2	95	1000	0.0403
2-95-2	95	2000	0.0378
3-95-2	95	3000	0.0333
4-95-2	95	4000	0.0256
5-95-2	95	5000	0.0246
1-100-3	100	1000	0.0349
<i>2-100-3</i>	100	2000	0.0320
<i>3-100-3</i>	100	3000	0.0270
<i>4-100-3</i>	100	4000	0.0184
<i>5-100-3</i>	100	5000	0.0172



# Traffic Considerations in Rigid Airfield Pavements

## Traffic Mix for Airfield Pavement Design



Must use the entire traffic mix, no more "Design Aircraft".

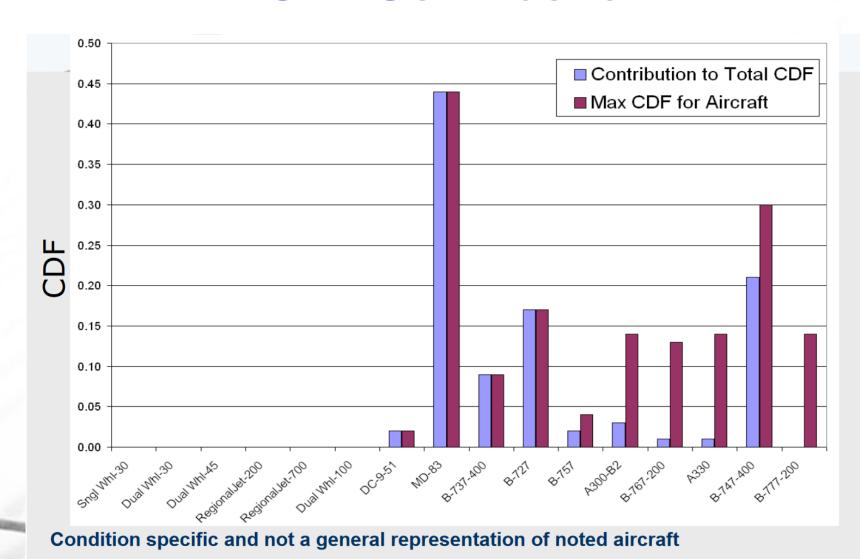
- > Traffic Model Cumulative Damage Factor (CDF) sums damage from each aircraft based upon its <u>unique pavement loading characteristics</u> and <u>Location of the main gear</u> from the runway centerline.
- > DOES NOT use the "design aircraft" method of condensing all aircraft into one design aircraft.

### Sample Aircraft Traffic Mix CDF Contribution

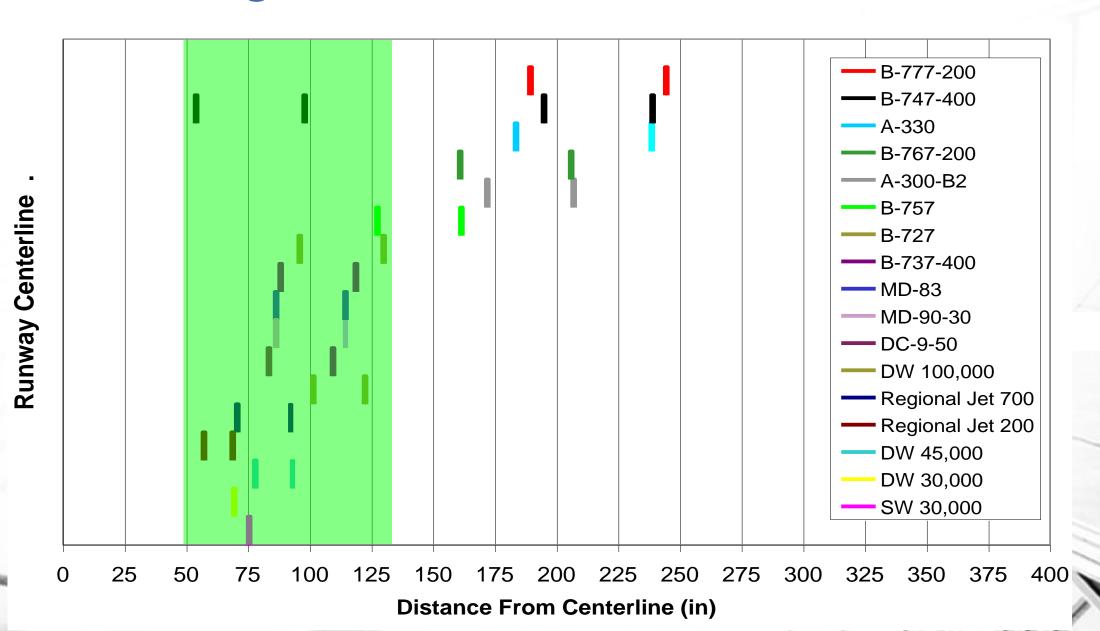
		Annual	CDF	CDF Max
Aircraft Name	Gross Weight	Departures	Contribution	For Aircraft
Sngl Whl-30	30,000	1,200	0.00	0.00
Dual Whl-30	30,000	1,200	0.00	0.00
Dual Whl-45	45,000	1,200	0.00	0.00
RegionalJet-200	47,450	1,200	0.00	0.00
RegionalJet-700	72,500	1,200	0.00	0.00
Dual Whl-100	100,000	1,200	0.00	0.00
DC-9-51	122,000	1,200	0.01	0.01
MD-83	161,000	1,200	0.39	0.39
B-737-400	150,500	1,200	0.09	0.09
B-727	172,000	1,200	0.23	0.24
B-757	250,000	1,200	0.02	0.03
A300-B2	304,000	1,200	0.01	0.16
B-767-200	335,000	1,200	0.02	0.15
A330	469,000	100	0.01	0.23
B-747-400	873,000	100	0.23	0.28
B-777-200	537,000	500	0.00	0.13
	<u> </u>			

Condition specific and not a general representation of noted aircraft

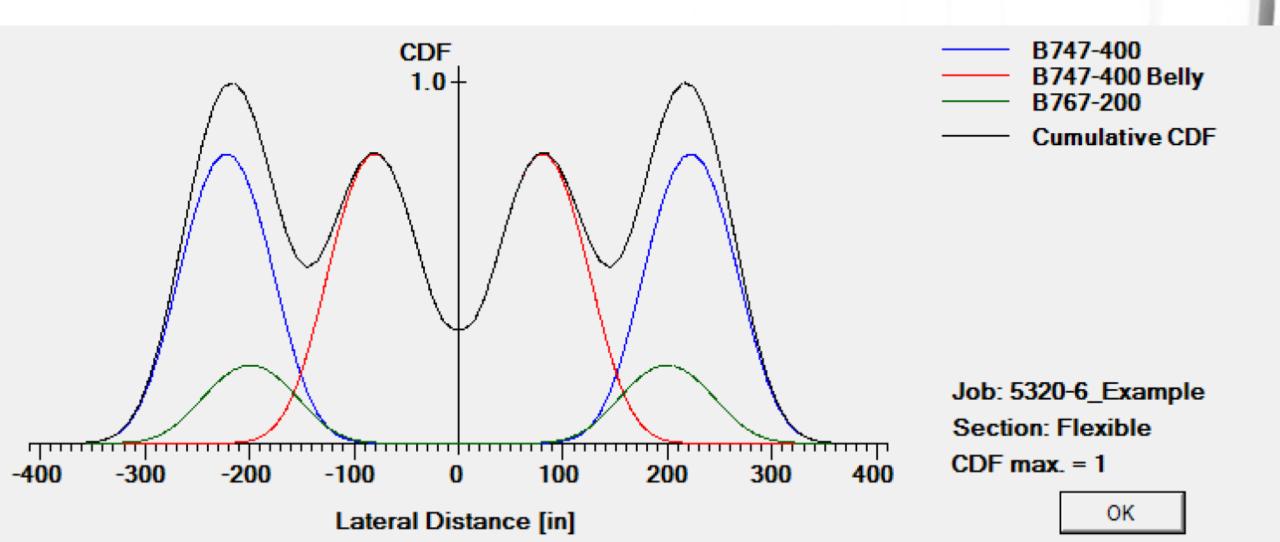
## Sample Aircraft Traffic Mix CDF Contribution



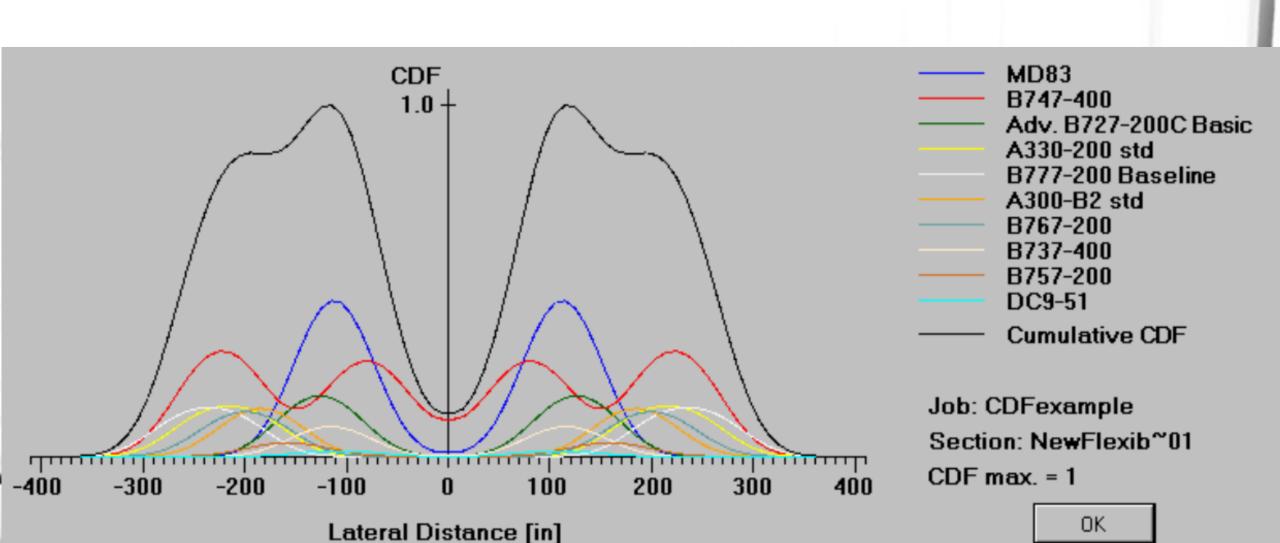
### Large Aircraft Gear Locations



### Cumulative Damage Factor (CDF) Graph



# Sample Cumulative Damage Factor (CDF) Graph



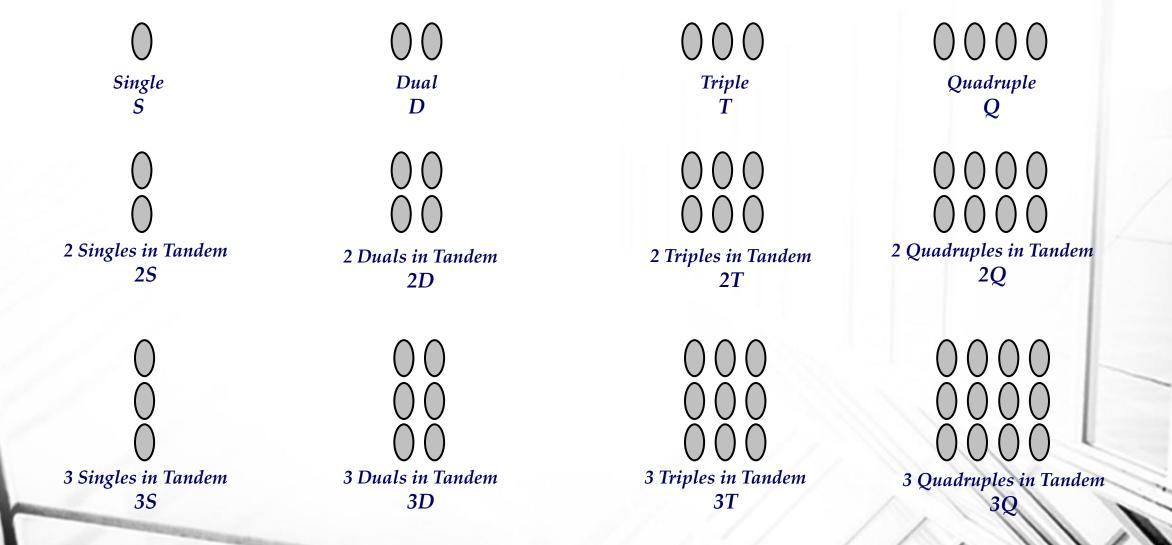
### Design of Airfield Pavement Shoulders

- > Shoulder must provide sufficient support for unintentional or emergency operation of any airplane in the traffic mix.
- > Shoulders are primarily intended to provide:
  - ✓ Protection from erosion and generation of debris from jet blast (no loose materials close to the runway).
  - ✓ Enhanced drainage.

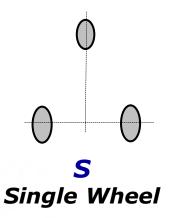


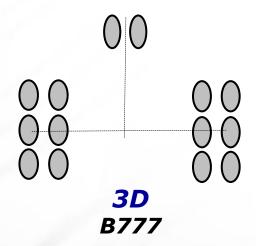


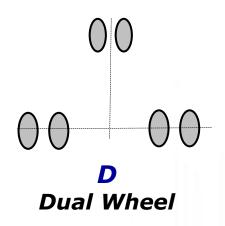
### **Gear Naming Conventions**

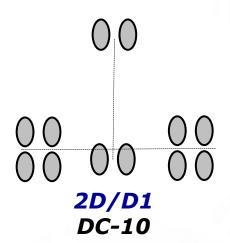


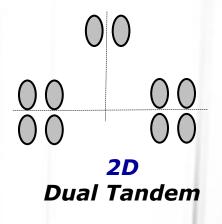
### Gear Naming Conventions- Examples

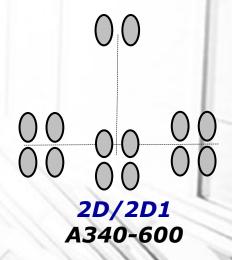




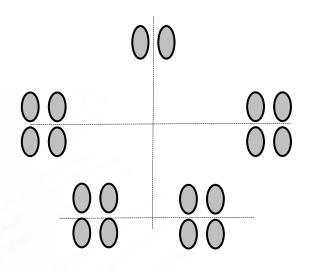




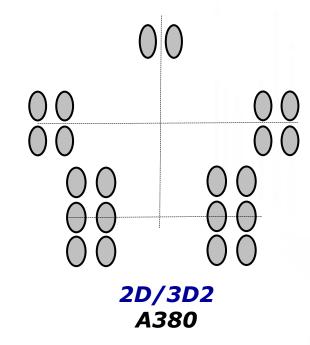


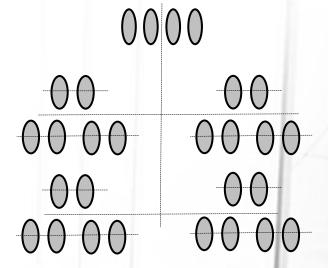


### Aircraft Gear- Examples

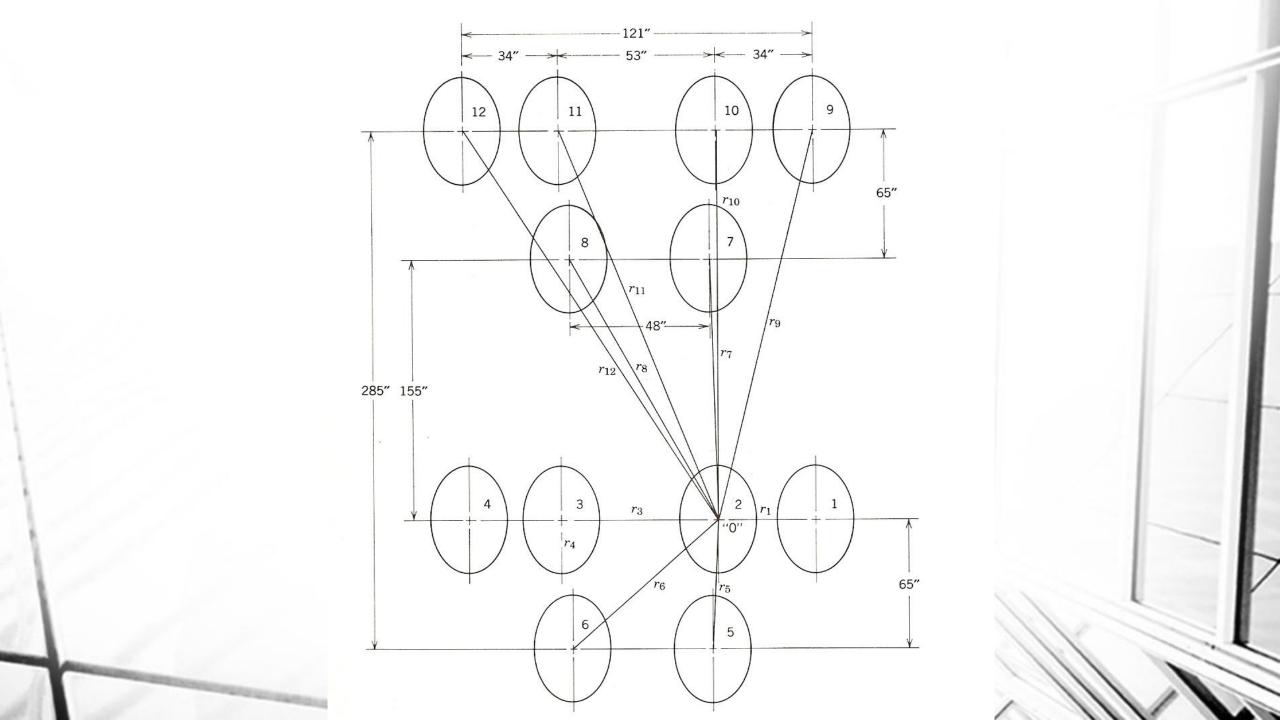


2D/2D2 B747





C5 Lockheed C5



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#### File Edit Help



-Input Layers

	Thickness	E-Modulus	PR	Slip	<u></u>
1	4.0000	0.60000E+06	0.45000	0.0000	≡
2	12.000	29000.	0.35000	0.0000	
3	0.0000	12000.	0.45000	0.0000	
4					
5					
6					+

-Input Loads-

	X-Coord.	Y-Coord.	Load	Contact Area	A
1	0.0000	0.0000	10000.	100.00	
2					
3					
4					
5					÷

-Input Evaluation Points

par	· c.1a c.1101111 011		
	X-Coord.	Y-Coord.	<b>A</b>
1	0.0000	0.0000	
2			
3			
4			
5			
6			+

Input Calculation Depths

	Depth	
1	3.9000	=
2	16.100	
3		
4		
5		
6		₩

-Results at Calculations Points

	Point 1	Point 2		À
X-Coord.	0.0000	0.0000		
Y-Coord.	0.0000	0.0000		
Z-Coord	3.9000	16.100		
Stress_X	-278.77	0.43036		
Stress_Y	-278.77	0.43036		
Stress_Z	31.761	6.9586		
ShearStress_XZ	0.0000	0.0000		
ShearStress_YZ	0.0000	0.0000		
ShearStress_XY	0.0000	0.0000		Ξ
Strain_X	-0.27936E-03	-0.24122E-03		
Strain_Y	-0.27936E-03	-0.24122E-03		
Strain_Z	0.47109E-03	0.54761E-03		
ShearStrain_XZ	0.0000	0.0000		
ShearStrain_YZ	0.0000	0.0000		
ShearStrain_XY	0.0000	0.0000		
Displt_X	0.0000	0.0000		
Displt_Y	0.0000	0.0000		
Displt_Z	0.22676E-01	0.15447E-01		
PrincStress_1	31.761	6.9586		
PrincStress_2	-278.77	0.43036		
PrincStress_3	-278.77	0.43036		
PrincStrain_1	0.47109E-03	0.54761E-03		÷
4			<b>&gt;</b>	

<u>C</u>alculate

<u>S</u>ave

<u>O</u>pen

Clear <u>A</u>ll

<u>R</u>eport

A<u>b</u>out

E<u>x</u>it



ERDC-WES Airfields & Pavements Branch

#### Problem 1.

Perform a sensitivity analysis of the modular ratios of two consecutive layers (e.g.  $E_{AC}/E_{Base}$ ) in a three layer system. Provide plots of vertical and shear stress distributions vs. depth for the following scenarios and discuss your findings.

- Assume the modulus of the base layer as your last 5 digits of university ID number and develop plots of vertical and shear stress distributions vs. pavement depth along the load centerline for E<sub>AC</sub>/E<sub>Base</sub> = 5, 10, 20, 30, 40, 50, 100.
- Assume the thickness of the surface layer as 4 inches, and the thickness of the second layer (base layer) as 12 inches.
- ➤ Consider a soft subgrade with modulus of 4,000 psi and a stiff subgrade of 12,000 psi for your simulations.
- Assume a tire pressure of 100 psi acting on the surface of the pavement; assume the tire contact radius of 5.5 inches.
- Assume any other parameter/material property that you might need for your simulations; provide a justification for your assumptions.

#### Problem 2.

A proposed pavement design under construction for a highway consists of 6 inches of Hot Mix Asphalt (HMA) layer with modulus of 500,000 psi, and 10 inches of unbound granular base (UAB) layer over a relatively soft subgrade with modulus of 5,000 psi.

Determine the number of 18,000 lbs. ESALs that this pavement structure can support to reach rutting and fatigue failure. Use both Asphalt Institute (AI) and Shell transfer functions and discuss your results.

- a) Vary the modulus of the base layer from 15,000 psi to 60,000 psi in 15,000 psi increments.
- b) Assume proper Poisson ratio for each layer when you analyze the pavement system in a layered elastic software (such as WINJULIA or KENLAYER).
- c) Assume the contact radius for the tire footprint on the pavement as 5.5 inches.
- d) Discuss the influence of base layer modulus on the critical pavement responses.